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Solow Growth Analysis: Further Analysis of the Model's Progression Through Time

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In the 1950's Robert Solow created a system of equations that would combine the input of labor and capital to generate a model of a country's economic path. Our objective is to take a closer look at the Solow growth model. We will analyze this system of differential equations to learn more about the growth of economies in both developed and emerging countries, and what factors play an important role in a country's economic welfare. In doing so we will be able to see stable and unstable solutions, characteristics of each model, and adapt the Solow model to provide more realistic results. We will collect archived data for different countries from the World Data Bank to use as coefficients for our modified differential systems and compare the resulting projections to historical data. We found that by allowing for more economic variables, the Solow Growth Model creates a more realistic path prediction.

1. Introduction

We present a descriptive analysis of the Solow growth model, also called the Neoclassical growth model, and how different rate parameters affect an economies path through time. The Solow growth model has many variations, including equations that take into account changes caused by increases in technology and the value of education to a country. These variations of the Solow growth model are our main focus. They will tell us more about how changes in education and technology affect a country's economy over time. With this information, we can see which countries follow similar paths. Our goal is to better understand United States economic fluctuations, and to compare them to the fluctuations of other countries.

2. Solow Growth Model

There are two equations that comprise the Solow growth model: the production function, PF, and the capital accumulation equation, CAE. The production function, PF, is the equation that displays the relationship of the inputs of an economy to the outputs. For the base case Solow model, we assume a Cobb-Douglass PF:

$$\mathrm{Y} = \mathrm{K}^{\alpha} * \mathrm{L}^{1-\alpha} \tag{1}$$

That is, output, Y, is equal to total capital, K, raised to some number α, between 0 and 1, multiplied by the input of labor, L, raised to the $(1 - \alpha)$. The equation exhibits constant returns to scale. This means that for every 1% change in the total input, the output given by the PF will also change by 1%. Choosing α is one of the most important initial steps when solving the Solow model, since α determines the shape of the function. We can estimate this parameter by fitting the resulting PF to historical data. The CAE is the base differential equation

(DE) for the Solow model. This DE gives the change in capital, *K*', in terms of *Y* and *K*. In this simple Solow model, there are two growth rates: the savings rate, *s*, and the capital depreciation rate, *d*.:

$$K' = sY - dK \tag{2}$$

The change in capital is gross investment, *sY*, minus total depreciation, *dK*. We can alter the PF and CAE to represent the model of our choosing, and use historical data from different countries to model the progression of their economic state over time. To see this development we must first select a starting year and enter the collected data into the equations above for all constant variables. Next, we must find data from the selected starting year to represent the variables that change over time; these will be our initial conditions, IC. The result will be a graph showing the path that the country's economy is expected to follow, taking into account changes in both labor and capital. This can be compared to the actual path depicted in the graphs in Appendix A. A more detailed graph of our results would be presented in the form of a phase portrait, which shows the steady state solution and how each point on the graph will move if our IC is chosen at that distance from the equilibrium. A steady state solution is defined as the equation that generates $K' = 0$. In other words, we find the parameters that, if plugged into K', cause capital to be constant over time. "The further an economy is 'below' its steady state, the faster the economy should grow. The further an economy is 'above' its steady state, the slower the economy should grow" (Jones 98). This is the theory behind the Neoclassical growth model. It assumes poorer countries should catch up to the wealthy, causing all to converge, and follow the same economic path. However, mitigating factors, both positive and negative, will cause one or both of the country's rate parameters to adjust. When this change happens the entire model will shift and become an entirely new graph. How do we account for these "shocks"? We can use the flexibility of the Solow model and create our own systems of differential equations to model how specific shock factors affect an economic path.

We select 8 of the countries that Charles I. Jones also investigated in *Introduction to Economic Growth*. We study 4 "rich" countries (France, Spain, United Kingdom, and the U.S.), and 4 "poor" countries (China, India, Uganda, and Zimbabwe). We collect the needed data from both the book and from The World Bank. The parameters that we needed to retrieve for our problems were the following: Adjusted savings as a percent of GNI, GDP per capita, GDP per person employed, labor force total, number of resident patent applications, population growth, and number of researchers in R&D. We will discuss later how each of these are incorporated in the Solow growth model we are studying.

3. Changing the α Parameter

There are three general cases we consider. The first is when α from equation (1) is equal to 1. It then follows that the total output would be equal to the input of capital since the PF would be $Y = K^1 * L^0 = K$. This means that, in this economy, output is only generated from the input of capital, and no labor is factored in. If this is true, then the CAE may be written solely in terms of *K*, taking the form:

$$K' = (s - d)K \tag{3}$$

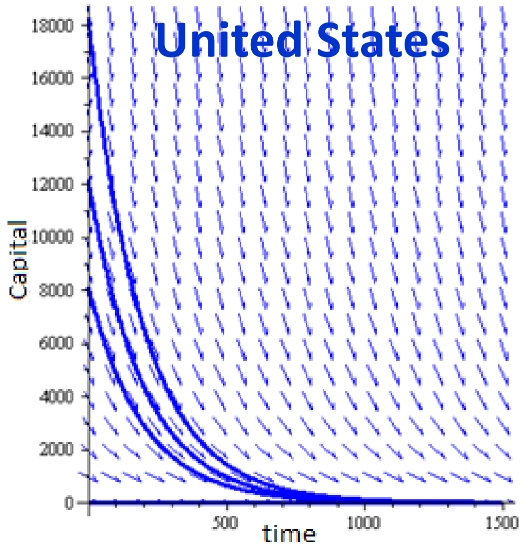
Within this case there are two variations that we will focus on: (a) both *s* and *d* are constants, and (b) when *d* is a constant, but $s = \alpha - \beta K$.

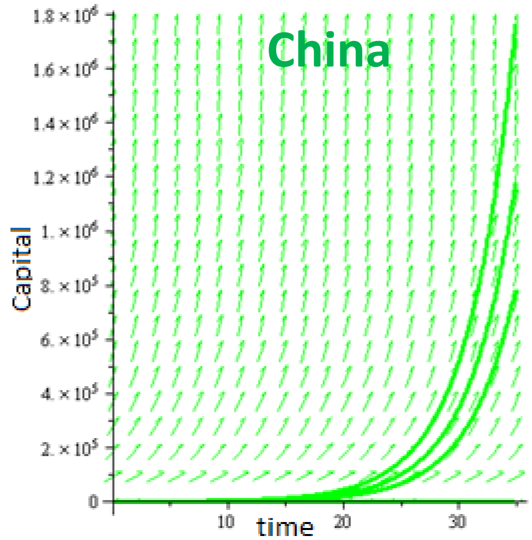
The first variation, where *s* and *d* are constants, would have a solution of the form

$$K = Ce^{(s-d)t} \tag{4}$$

The C and (*s*-*d*) in this solution are constant. From now on we will assume the *d* for each country is .05, since this is the rate of depreciation that Charles I. Jones used in his book. The only variable would be time, t. If $s > d$, then over time the amount of capital in the economy will increase. Conversely, if $s < d$, the capital will deplete to zero and the economy will fail. This is plausible because, with a depreciation rate less than savings rate, a country with this characteristic will save faster than depreciation can take value away. Also, with more savings there is more capacity to reinvest in capital, and since the PF is derived from capital the economy will flourish. If *d* is bigger than *s*, over time capital will deplete faster than the rate at which the economy saves and the country will not have the ability to continue at this low state. We choose to represent the savings rate with adjusted net national savings as a percent of GNI, archived in the World Data Bank from 1970-2009 because, in "Introduction to Economic Growth", it states that the American savings rate was .056 and when we average the data, the rate was approximately .0559. As we see, the American rate we use is less than this. This is because we need to shorten the time interval the data is averaged over because our data set didn't start recording the

GDP per person working until 1980, and we wanted to keep it consistent throughout all the examples. Therefore, we adjust the time line to be 1980-2009. The phase portraits shown below are examples of the properties described above. For more examples, all graphs are displayed in the appendix. American $s = .04489573842$, making the $s < d$, while in china the $s = .2989782231$, $s > d$. The United States economy is decreasing at an exponential rate to an equilibrium, while China's economy is increasing exponentially. The arrows displayed on the graphs represent the direction the country's economy moves along the solution curves.





China's path increases with no equilibrium because without any outside factors if savings is greater than depreciation there will be no limit to the growth. Conversely, the U.S. levels off because, after almost 1,000 years of depreciation outweighing savings, the economy would run out of savings to invest in capital, and the resulting economic state would be at zero. The IC's used for these graphs were the GDP per capita from 1980 for each country. The DE would be at a steady state solution, if: $s = d$, or $K = 0$. The first condition makes $K = C$, and the second can only occur when $C = 0$, since K is a constant multiplied by an exponential. This model does not seem to be a good representation of an economy because the graphs look identical, which should not be true. Since an economy's structure is built from many contributing factors, we would assume that the model of an economy would have more freedom in its movements. Also, some of the forecasted economies, such as Zimbabwe, will converge to zero after a very low number of years. The phase portrait for Zimbabwe, as shown in the appendix part 1a, suggests that after 500 years the economy will have no capital left and will stay that way.

For the second variation (b) from equation (2) we chose to explore when the savings rate is governed by the amount of capital. One way we can represent this is to assume savings is $s = \alpha - \beta K,\ 0 < \beta < 1$. When looking at the phase portraits we noticed that if the value of β was close to 1, the slope of the models were unreasonably steep. Therefore, we arbitrarily chose β for this system to be .0002 to try and model the economic path as best we can. Under these circumstances, together with an α of 1, the CAE would be:

$$K' = [(1 - \beta K) - d]K = [(.95 - .0002K)]K \qquad (5)$$

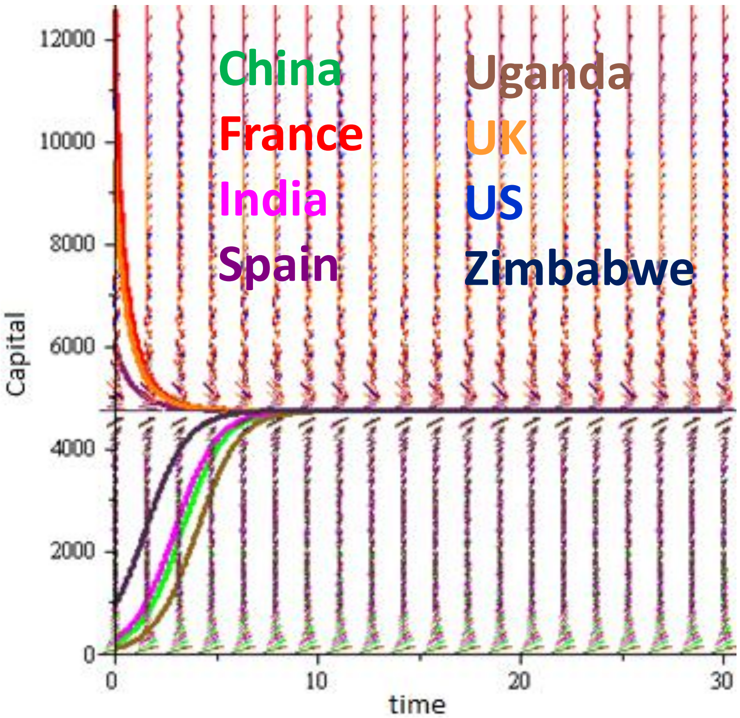
The following equilibriums would create a steady state solution:

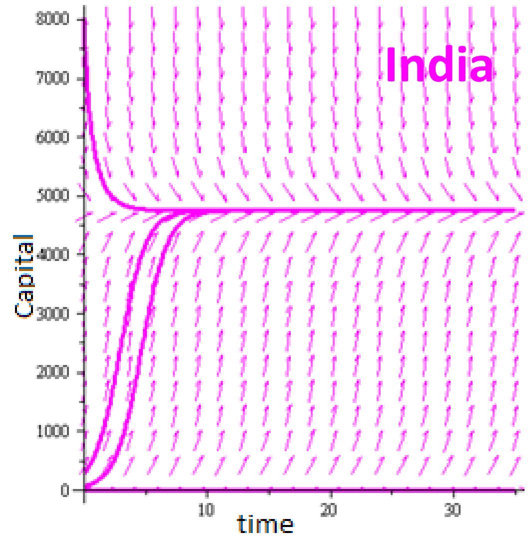
$$\mathrm{K} = \frac{1-\mathrm{d}}{\beta}, \text{ where } \beta \neq 0, \quad \text{and K=0} \qquad (6)$$

The solution to this DE is:

$$K = \frac{d-1}{e^{(d-1)*t} * e^{(d-1)*C} - \beta} \qquad (7)$$

The graphs of this solution are a perfect example of the Neoclassical growth theory. If the IC of a country started above the equilibrium the capital would decrease, while the IC's below the equilibrium would increase. The phase portraits for this system all converge to the same equilibrium, which is just under 5,000. Shown below are samples of the structure the model creates for the economies. The first graph is to show all the countries in comparison to each other, while the second is to show the typical form the graphs render. Again, we used GDP per capita for 1980 as the IC, $\beta = .0002, d = .05$, and the s corresponding to each country to form these graphs.





One fact that should be pointed out is that when an IC was chosen above the equilibrium, the path would decrease at a faster pace than the slope resulting from an IC below the equilibrium. This model also does not seem to be a reasonable fit to a real economy because it suggests that the path will be monotonic, and will increase or decrease based on whether the IC is above or below an equilibrium. Also, shown in this picture, all countries converge in less than 15 years and since the data used started in 1980 we know that this did not happen.

The second case we will examine is when α from equation (1) is 0. If this is true, then $Y = K^0 * L^1 = L$ and the CAE would be, $K' = sL - dK$. This implies that the output in this economy is derived directly from labor, and capital arises from savings of labor minus the depreciation of capital. Because we are adding a new variable to the calculations, L, we must know the rate of change of labor, L', and find the solution for both DE's. Within this case we will focus on the following three variations:

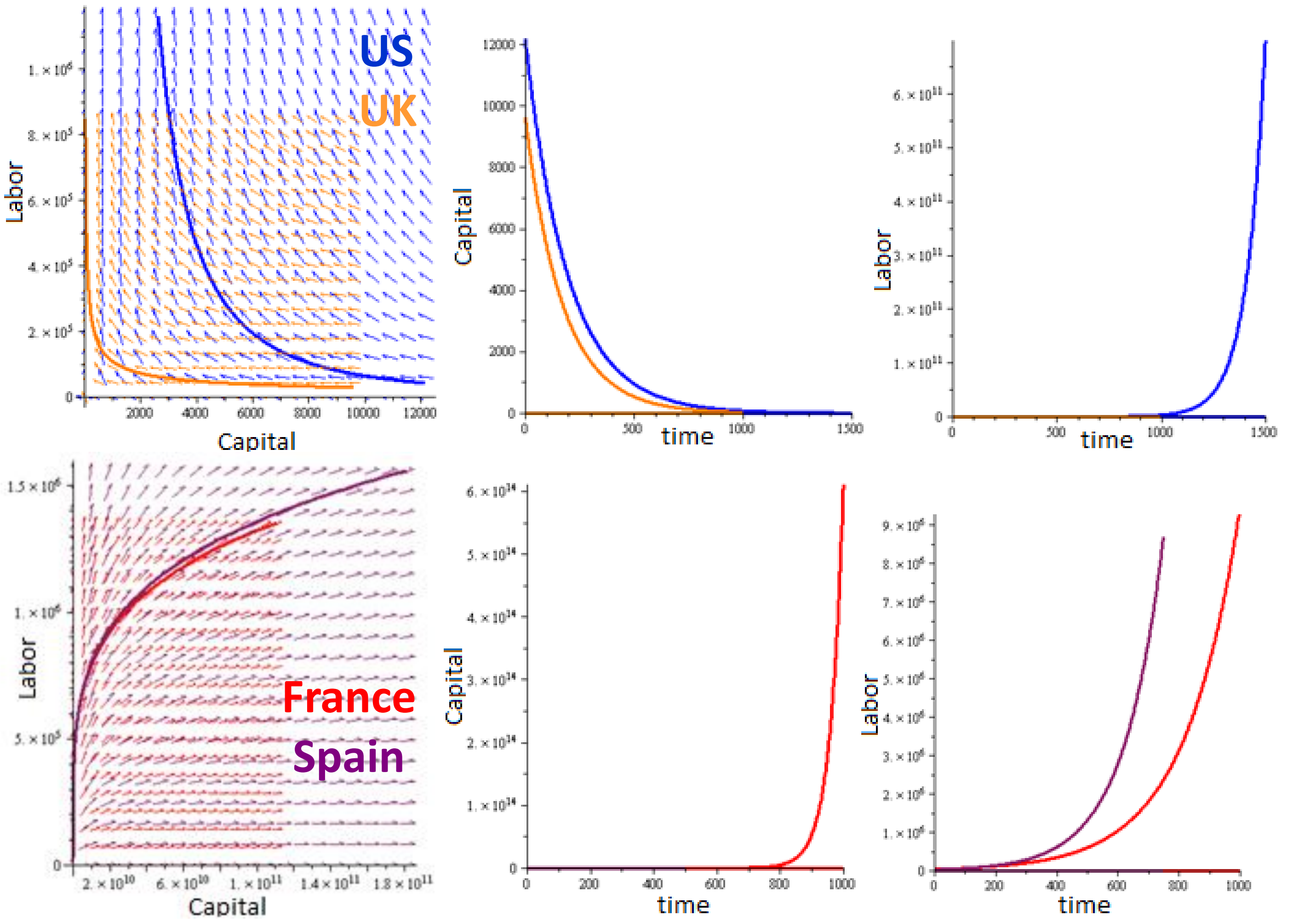
$$\text{(a)} \quad L' = n * L\,, \quad \text{(b)} \quad L' = \frac{\alpha - \beta * L + \gamma * L^2}{\delta * L}, \qquad \text{(c)} \quad L' = \frac{\alpha - \beta * K + \gamma * L^2}{\delta * L} \tag{8}$$

We choose this fractional structure to give the equations more flexibility for a general case as well as for the form of their solutions. For the β, ϒ, and δ parameters we will arbitrarily choose these values to get the best graphical representative of an economy. Their values are based on the viability of the graphs produced under those conditions.

The first, case a, is the assumption that labor force grows at some constant rate *n*. If this is true, then $L' = nL$. As in Jones' *Introduction to Economic Growth*, we will assume, for this case, that the rate *n* is the same rate as population growth. Because of this, when we run simulations the *n* will be the average population growth for a specific country over the time period that we are studying. Therefore, if the population increases by 1% then the labor force also increases by 1%. There will be an equilibrium solution at $K = 0,$ and $L = 0$. The solution to this problem is when

$$L = L_0 e^{nt}\,, \tag{9}$$

where L_0 represents the IC, which will be the GDP per person working from 1980, and e^{nt} yields the new labor force total when multiplying it by that IC. This is plausible because if your output is generated solely from labor and the population increases over time then your output will also increase at the same rate. Similar to the first problem in section 1, when $s > d$, the path of the country's economy will be increasing, and when $s < d$ it will be decreasing. The equilibrium solutions are when, $L = 0,\ K = 0,$ and $s = d$. Since the solution is in exponential form, the phase portrait is also much like those in section 1.

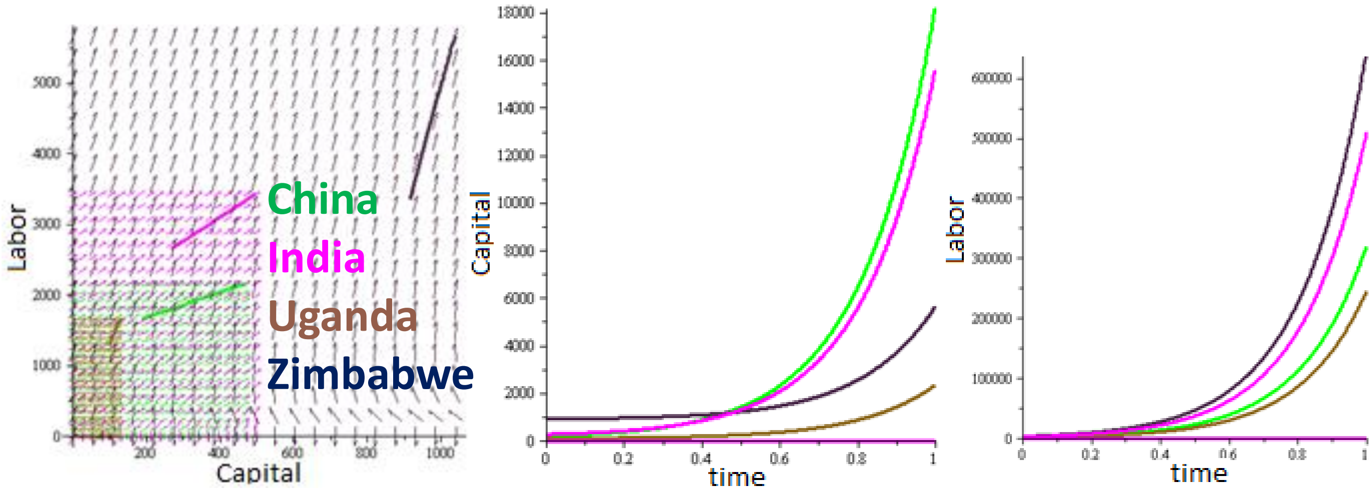


This model does not seem to be a suitable fit, as they do not converge to any equilibruim. There are still only two shapes created by this model and, again, they are based on only the two parameters, *s* and *d*. Had this been a true model of these economies, the American and United Kingdom paths are more efficient because if output is only produced from labor it wouldn't be logical to increase capital as France and Spain do.

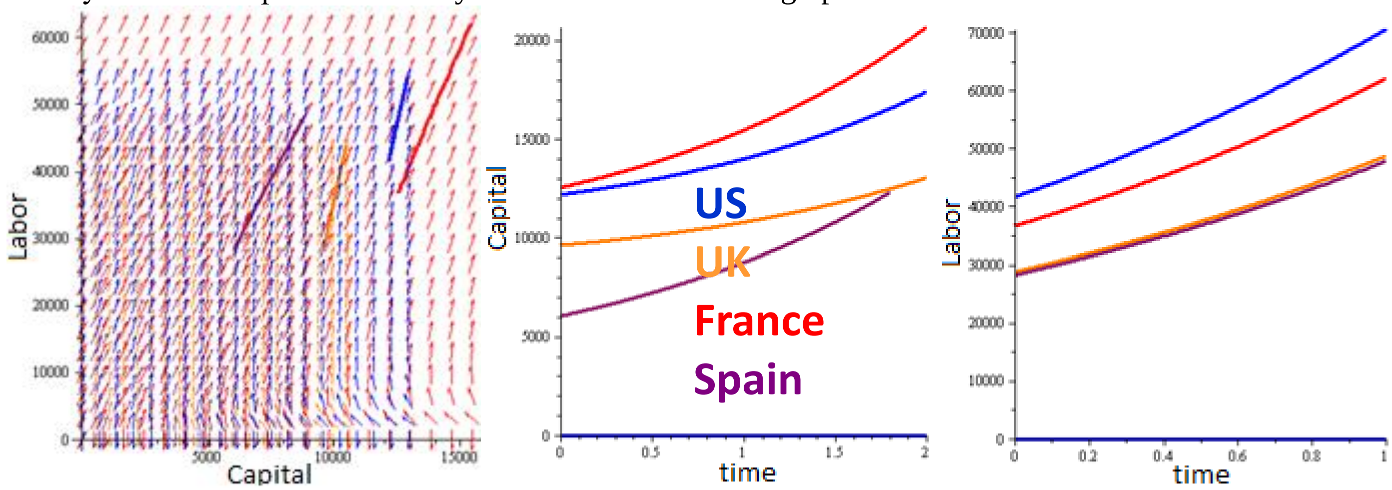
If growth is not constant, there are many alterations that could be made to show how the labor changes through time. Again, we will be looking at equations (8b), Case b., and (8c), Case c. The equation in Case b. was chosen to have L' be governed only by L, but for Case c. we wanted to see how letting L' depend on K as well would change the shape of the model. When drawing the graphs for Case b., if we choose a Υ that is too small the country's paths travel at a slope abnormally close to 0. So, for this system we will choose Υ to be .02. Since δ is in the denominator, small values of δ cause computational problems for Maple and the graphs show negative values for both the labor and the capital. Because of this we will make $\delta = .0095$. Also, to keep constant with the previous problems we will also choose $\beta = .0002$. The equilibrium from Case b. would be:

$$L = 0 \tag{10}$$

The phase portraits for this equation do not converge to any equilibrium. With $\propto = 0$, L' would be increasing or decreasing at a constant rate, causing labor to move monotonically. Below are some examples, and as $\gamma > \beta$, the function will always be increasing.



For Case c., the equilibrium solutions are when K=0, and L=0. The difference between this solution and the previous, where L' doesn't depend on the amount of capital, is that the countries from Case b. grow exponentially, while the paths the same countries take in Case c. are much more gradual. They both, however, are only functional over a very short period of time. There are some computational errors introduced in some country's economic paths after 1-2 years. Illustrations of the graphs from Case c. are shown below.



In the next section we look into the more general case, when $0 < \alpha < 1$, and change sections of the base case equation to include more model changing factors.

4. Education and Technology in the Solow Model

Now we will discuss the Solow model when we introduce the concept of changes in technology. A technological change is considered an important economic factor that could alter the original path created by previous parameters. We will assume A is the number of ideas that evolve into new technology and use the number of resident patent applications as the IC, retrieved from the World Data Bank. There are three production function options to choose from to find the solution to the model. The three choices are as follows;

Hicks-Neutral Technology: $Y = AK^{\propto} * L^{1-\alpha}$, (11)

Solow-Neutral Technology: $Y = (AK)^{\alpha} * L^{1-\alpha}$, (12)

Harrod-neutral technology: $Y = K^{\alpha} * (AL)^{1-\alpha}$. (13)

We will continue to follow Solow's work and use equation (12) the Solow-neutral technology production function. The PF will be $L' = g * L(t)$, and new CAE will take the form $K' = sY - (n + g + d)K$. Again, *n* is population growth rate, *g* is technological growth and *d* is the depreciations rate. This means that now the change in capital is the total savings from investment minus the increase in cost due to a change in labor force per capital, minus change in capital due to technology, minus the depreciation of capital per worker. We add the $(-g * K)$ term because the more advances there are, the less capital will be needed to produce the same amount.

Since we have three variables that change over time, K(t), L(t), and A(t), we will need three DE's. Therefore we will also assume that

$$A' = \delta * \rho * L(t)^{\lambda} * A(t)^{\varphi}, \tag{14}$$

the equation from Jones' *Introduction to Economic Growth*, where δ is the constant rate of change of ideas, and is calculated by $\Delta A = \delta = \frac{A(t)-A(t-1)}{A(t-1)}$. The parameter ρ will be the percent of the labor population that is involved in research and will be calculated by taking the number of researchers and dividing it by the total labor force. We will estimate λ, which accounts for the overlap that may occur in research projects. For example, if there are two or more firms working simultaneously to produce the same piece of technology, the time and energy that one of those firms could spend researching something else is wasted. Finally, φ accounts for the advantage that researchers currently have because of the technology that has already come into existence.

Lastly, we will introduce the value of education to the Solow model. The more a person is educated, the faster or more efficiently they are expected to produce. There may, however, be a point where the additional time spent learning instead of working will be outweighed by the value of experience. In this situation, it would be better suited for the individual to stop education and become a worker to gain experience. Also, if the education in a country is not up to par, their economy may suffer. The model we are introducing was created by Robert E. Lucas, Jr.(1988). In his version of the Solow model, there are new variables and equations.

$$Y = K^{\alpha} * (hL)^{1-\alpha} \tag{15}$$

"where h is human capital per person. Lucas assumes that human capital evolves according to

$$h' = (1-u)h, \tag{16}$$

where u is time spent working and 1-u is time spent accumulating skill."(Jones 98) We will be using and adapting this model to fit with the one we are creating. As in *Introduction to Economic Growth*, we will assume that u is the average number of years a person spends educating themselves, and that H is skilled labor, calculated as $H = e^{\psi * u} * L(t)$, where ψ is the percent increase in wages from every additional year in school. We will use the u's that are listed in the appendix of *Introduction to Economic Growth*, as well as use the same assumption, that $\psi = .1$. Now it is important to list all the equations we have just described to get a better look at the system we've generated.

$$Y = (A(t)K(t))^{\propto} * (L(t) * H)^{(1-\propto)} \tag{17}$$

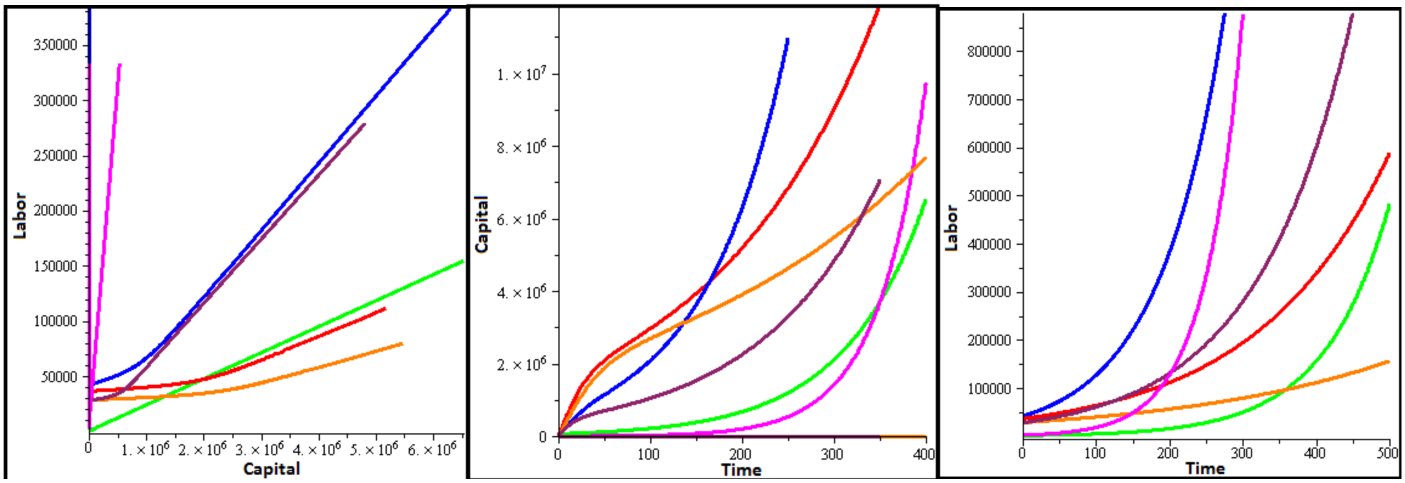
$$H = e^{\psi * u} * L(t), \quad where\ \ 0 < \psi < 1 \tag{18}$$

$$K' = s * Y - (n + d + g) * K(t) \tag{19}$$

$$L' = n * L(t) \tag{20}$$

$$A' = \delta * \rho * L(t)^{\lambda} * A(t)^{\varphi} \tag{21}$$

The parameters for the phase portraits of this new system seem to be insignificant. Whether φ was .0000001 or .8 the graphs were essentially the same, economically and financially. Because of insufficient data Zimbabwe and Uganda could not be modeled. For the ones that could be examined, the data used was from 1995-2009 since the number of patents was not recorded until then. An important characteristic to point out is that they take on different shapes depending on the IC. They seem to move monotonically in the same way, yet they all still have slightly different slopes and forms from each other. This system seems to be the best representation because, although they only increase, the graphs seem to move like you would expect an economy would, seemingly unpredictable. Some will be moving in the same direction and then diverge from each other. With more research and data on mitigating factors, a better system could be created using the same techniques. Another possible case to explore is if L does not grow at a constant rate. This may bring our model closer to portraying true economic movements more successfully.



China **France** **India** **Spain** **UK** **US**

5. Conclusion

A reoccurring pattern in the cases we examined was that capital and labor were always constantly growing. The only difference in the models is the rate at which they increased. Most of these fluctuations are actually exponential. Granted some of these growth projections predict thousands of years from now, however, our previous assumption was that the economic growth would at least have some of the same properties the Neoclassical model described. The only models that had decreasing capital was the first case, which did not take into account the change in labor, and the one with labor growing at a constant rate. This suggests that with labor and capital changing they will always be increasing unless some outside factor pulls them down.

We expected some countries, most likely the poorer ones, would have an inefficient level of education which would pull the labor and capital lines down. Without proper education the amount of capital would suffer. Also, we expected the technology gap to be another negative factor in the economy. More wealthy countries have the resources for R&D that most poor countries do not. The lack of technology was expected to hinder the less fortunate countries and level their capital and labor lines. A theory that could explain why this was not depicted in the graphs is because the poor countries utilize the technological advances of the wealthy without the expense for R&D, causing them to be more well of then they originally would have been.

We could also look into the fact that we may have overlooked an important factor that would cause these lines to decrease, or level off. Some of these factors could be the possibility of war. If we could somehow get the percent chance of war for each country we could include it in our prediction method. Another option would be to investigate limited resources. When resources are low, less labor and capital is used pertaining to that source. For example, if a forest was being cut down to produce chairs and there were limited trees, we might slow production, use less workers and machinery to allow for the resource to replenish itself. Finally, an economy is hugely impacted by natural disasters. The earthquakes in Japan are a prime example of this. Many resources will be put into rebuilding this economy instead of the normal production distribution. “In Japan, the 1995 Great Hanshin earthquake struck directly beneath the modern industrialized urban area of Kobe. It killed more than 6,000 people and resulted in an estimated $100 billion in damages, or about 2 percent of Japan’s gross national product (scawthorn *et* al., 1997)” (Chung and Okuyama). If we gathered all the information on natural disasters for each country and applied it to our model perhaps this extremely influential factor would help level the economic path.

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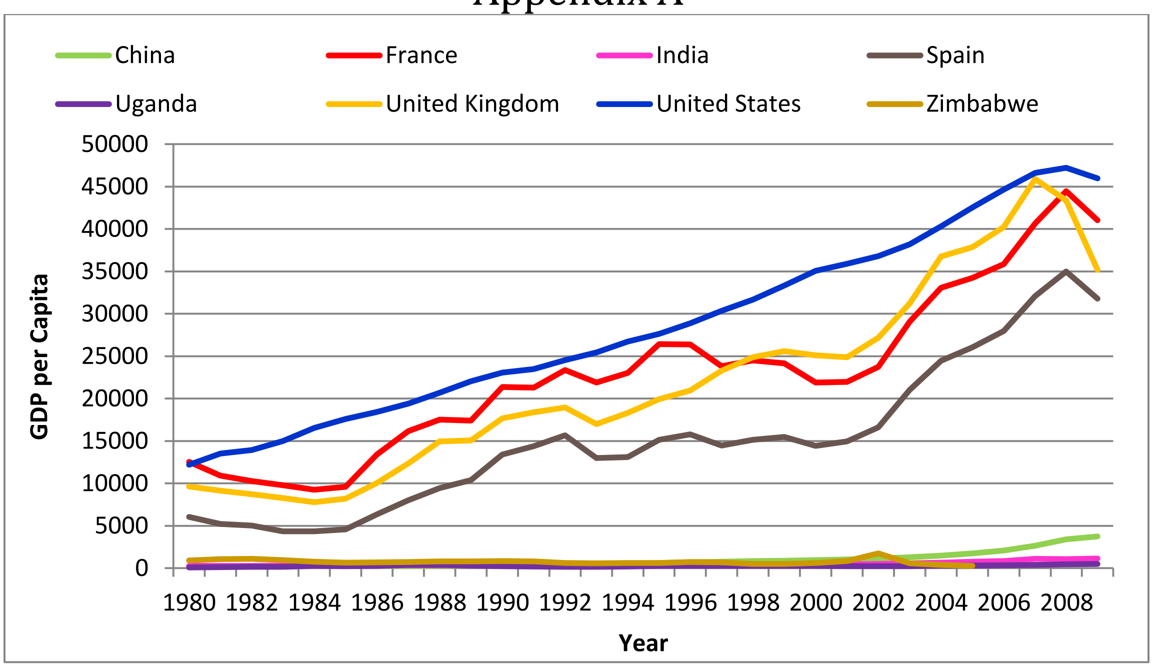
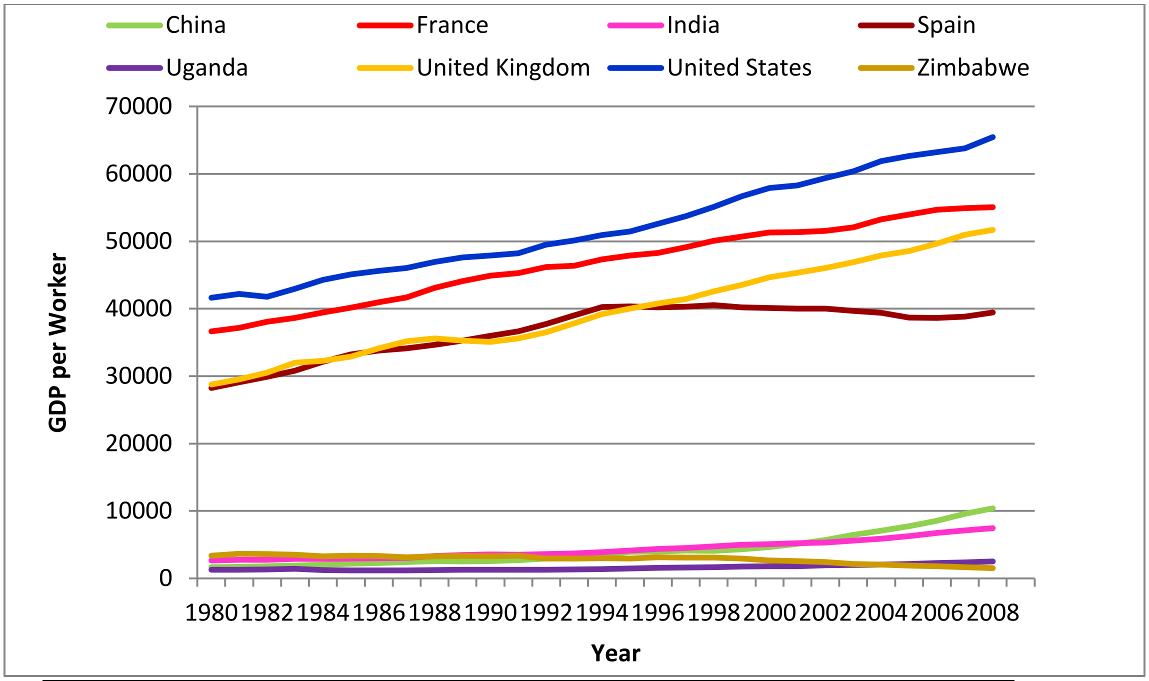
Appendix A

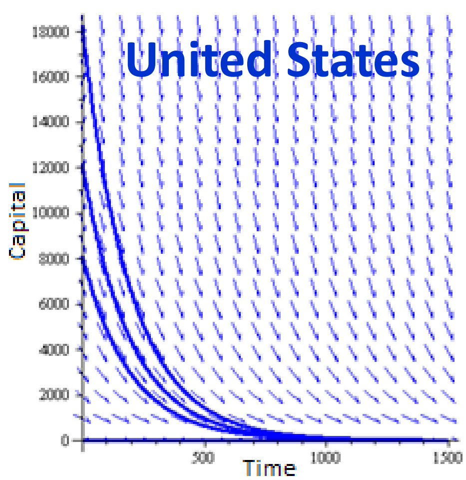


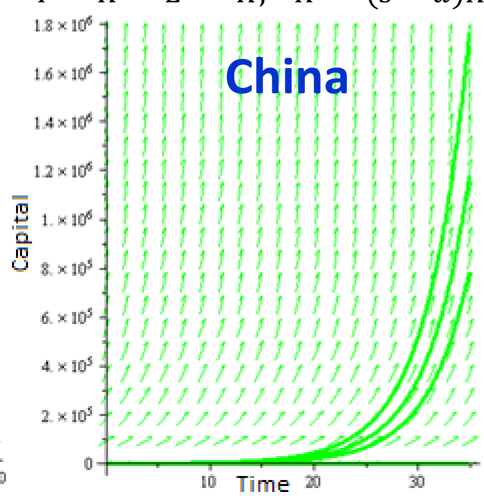


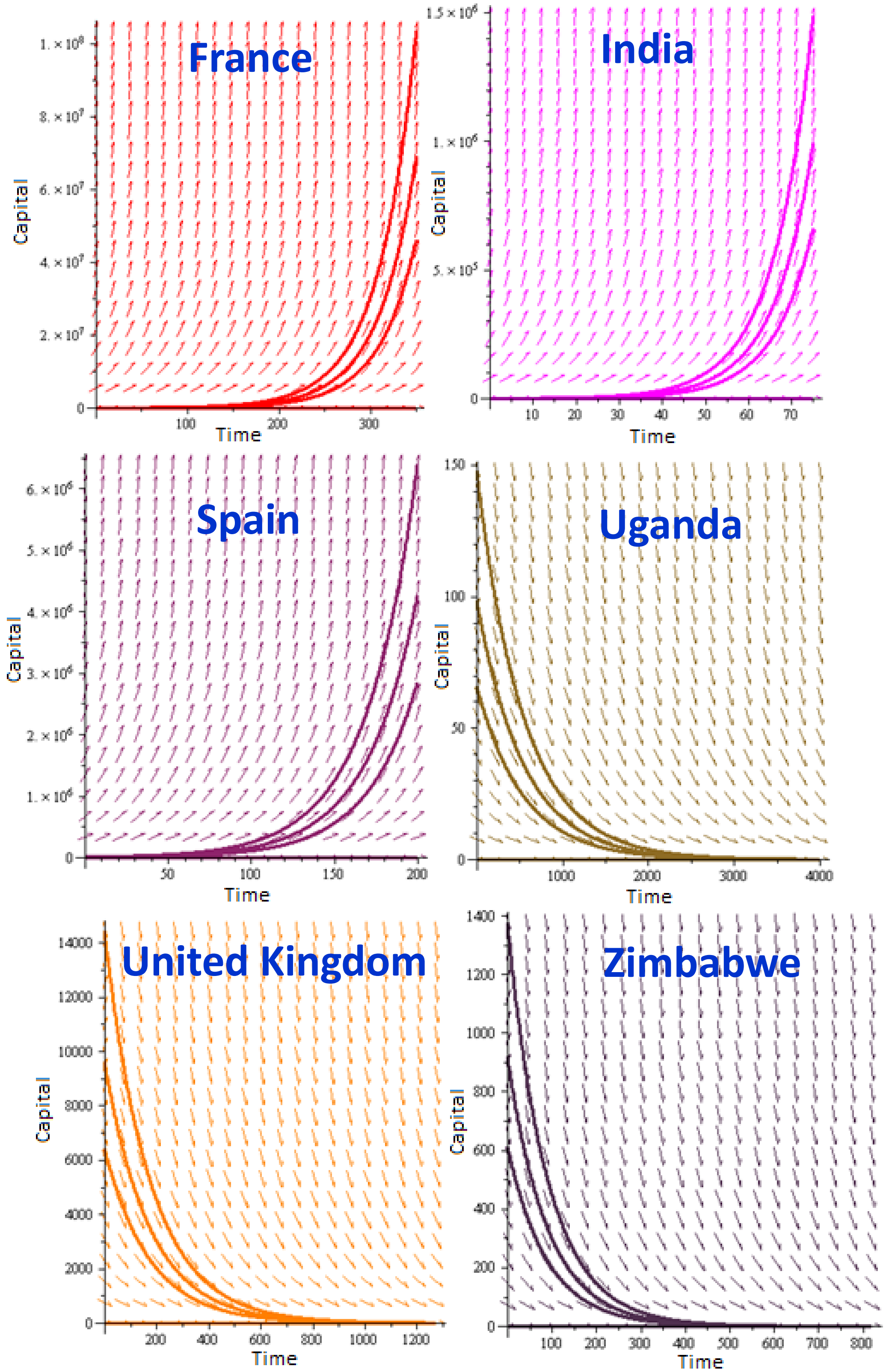
1980	Savings rate	GDP per Capita	GDP per worker	population growth rate
China	0.29897822	193.0220513	1655	0.011348628
France	0.07460781	12541.65079	36653	0.005548844
India	0.15955667	267.4087839	2638	0.0193524
Spain	0.08279544	6045.136464	28226	0.007638864
Uganda	0.04816636	98.34619162	1268	0.034994166
United Kingdom	0.04420771	9622.974872	28753	0.003384596
United States	0.04489574	12179.55771	41649	0.011090205
Zimbabwe	0.03878701	917.1316771	3344	0.020640785

Appendix B

$\propto = 1$, s and d=.05 are constants: $Y = K^1 * L^0 = K, \quad K' = (s - d)K$






France
Capital
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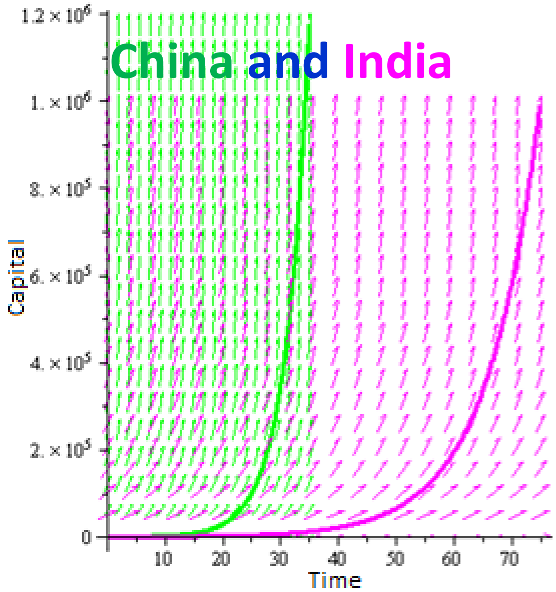
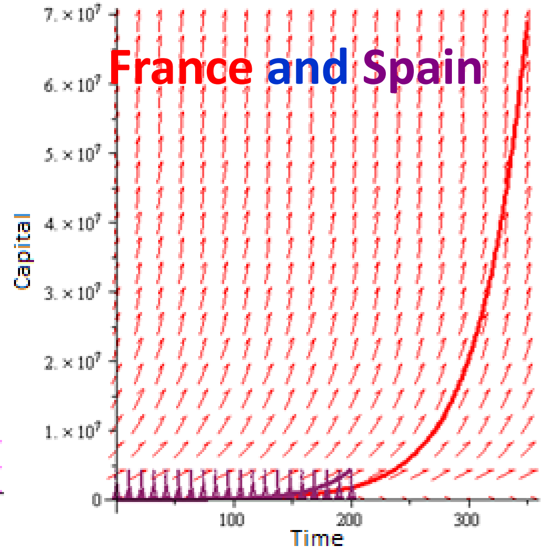
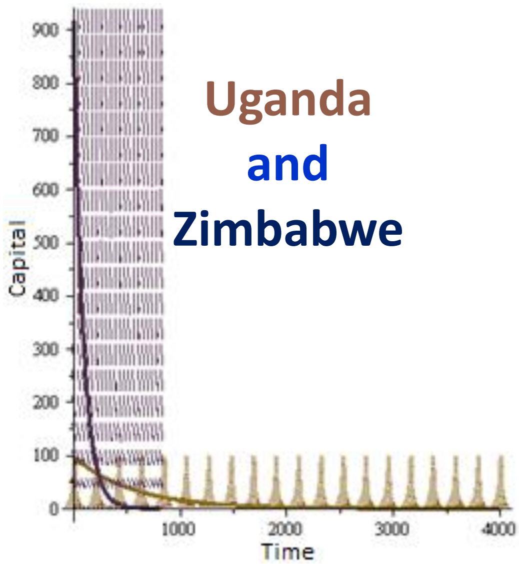
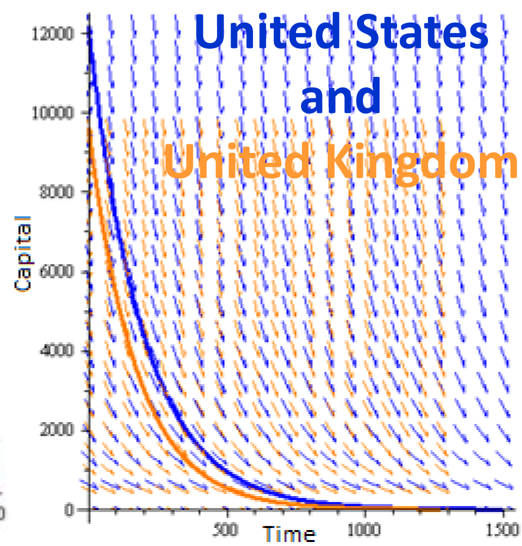
800



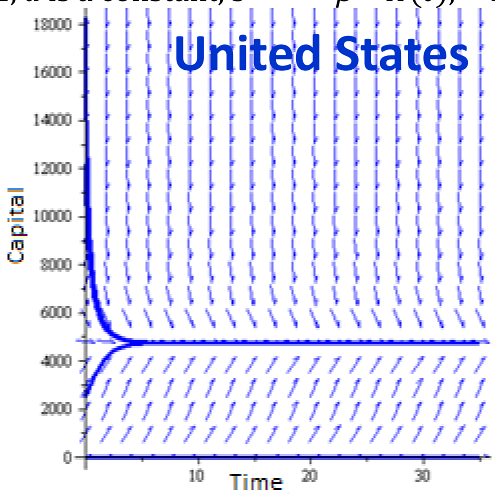


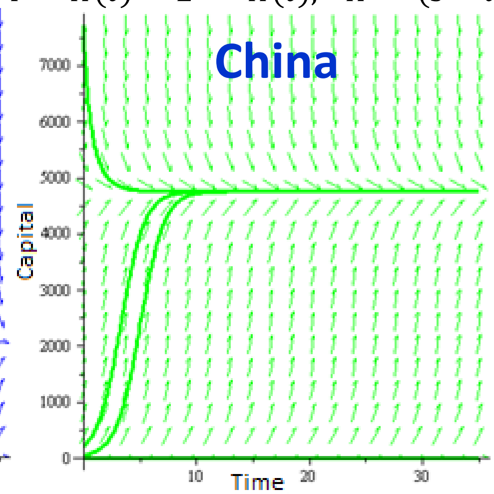


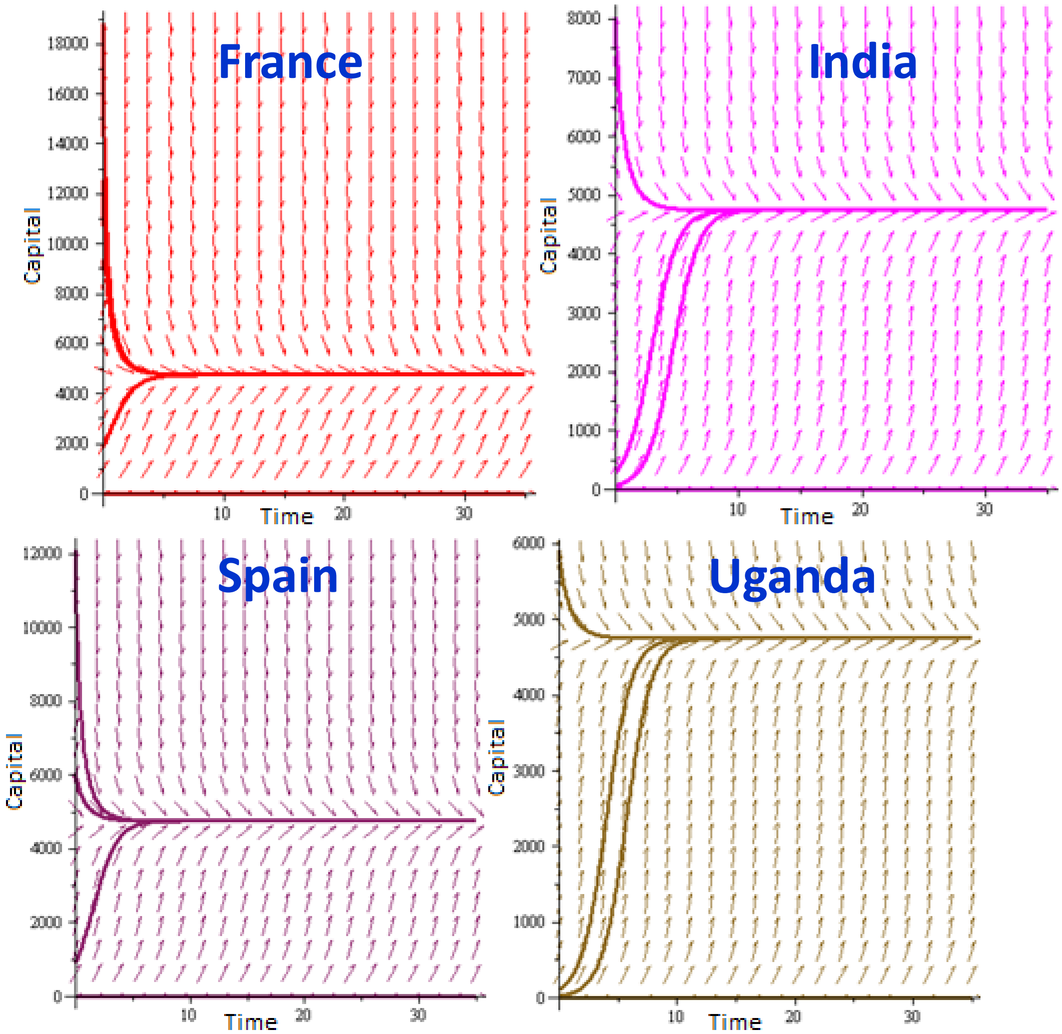


Appendix C

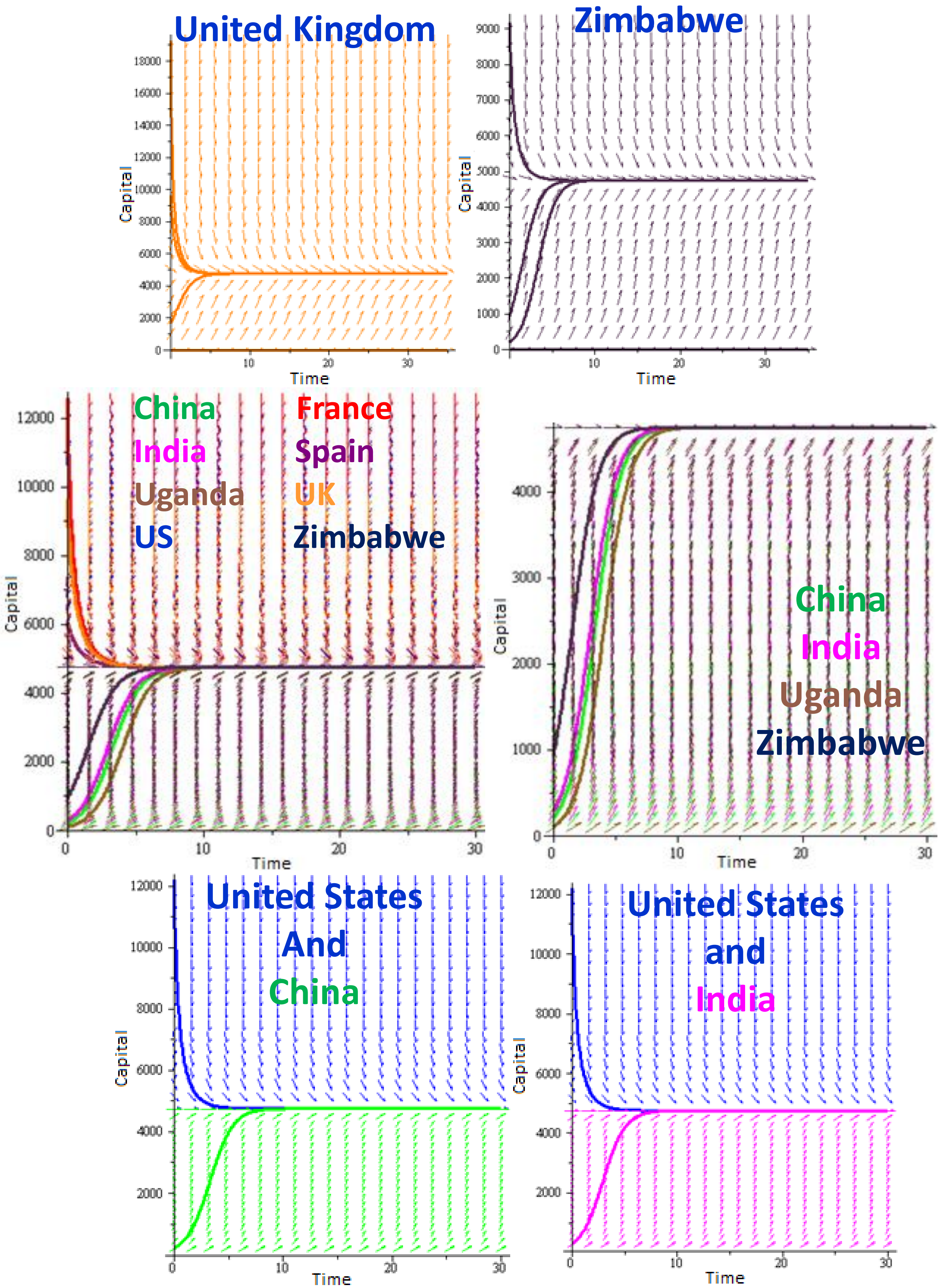
Part 1b) $\alpha = 1$, d is a constant, $s = \propto - \beta * K(t)$, $\quad Y = K(t)^1 * L^0 = K(t)$, $\quad K' = (s - d)K(t)$







France
India
Spain
Uganda
Capital
Time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China
France
India
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Uganda
UK
US
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China
India
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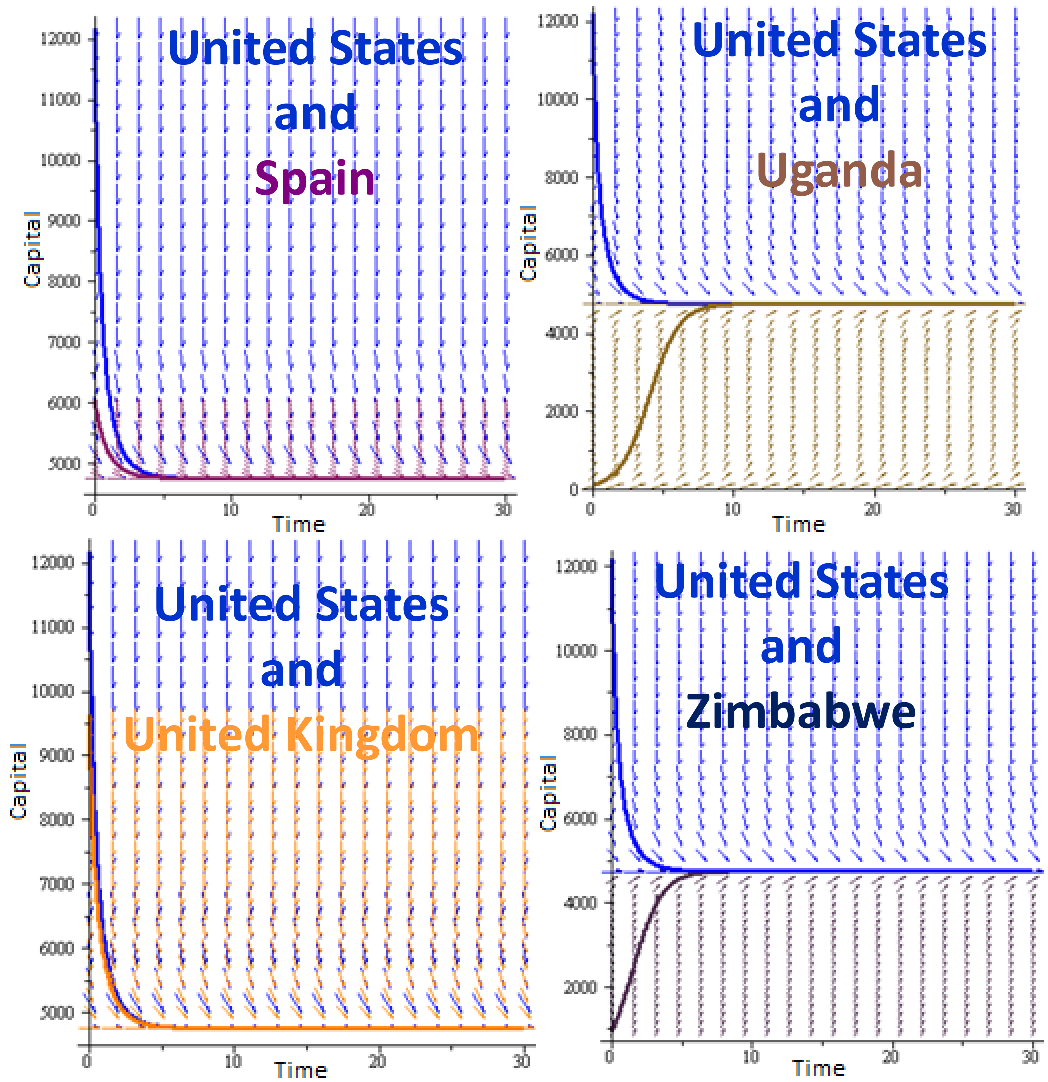
Zimbabwe
United States
And
China
United States
and
India
Capital
Time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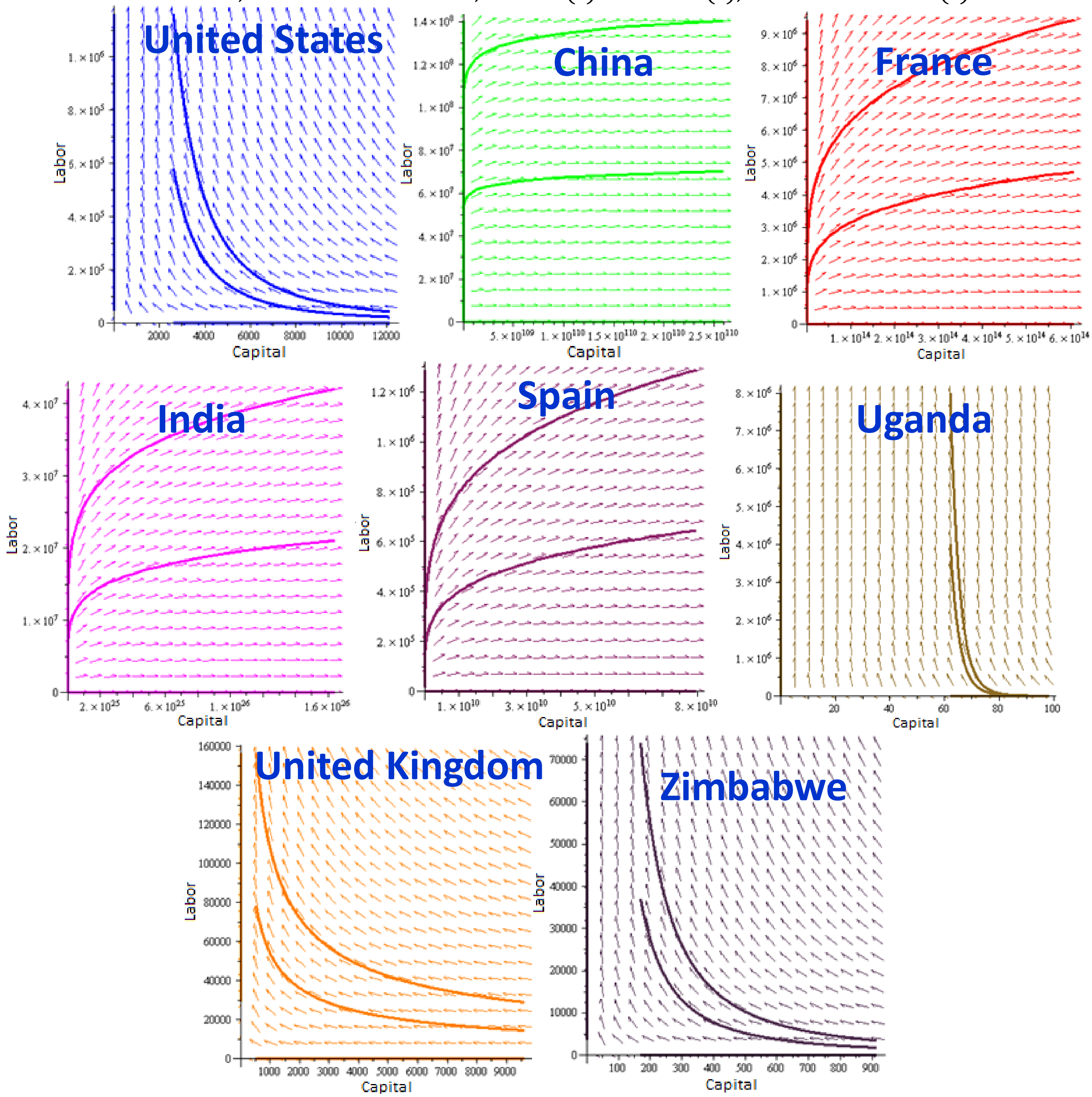
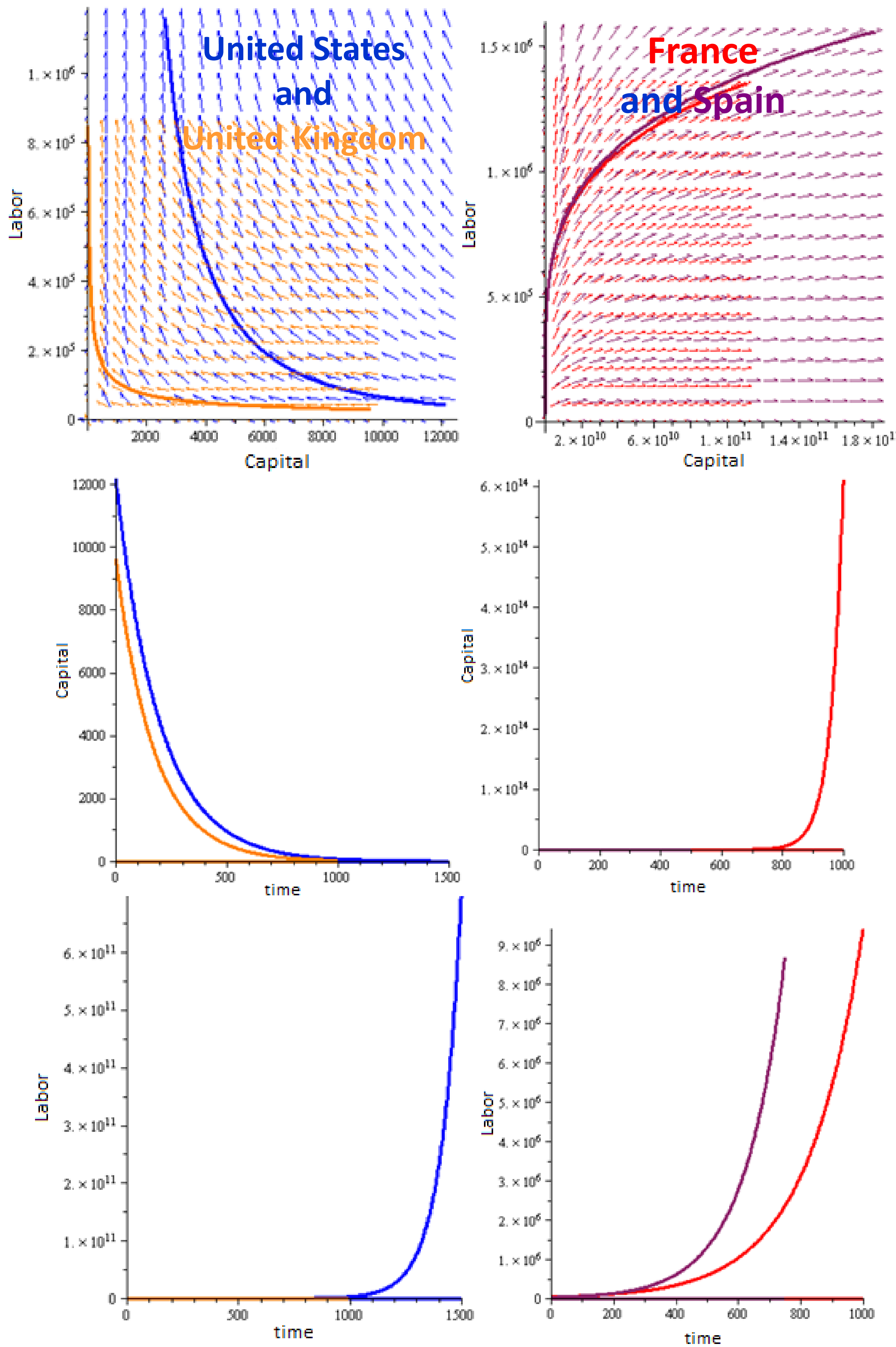
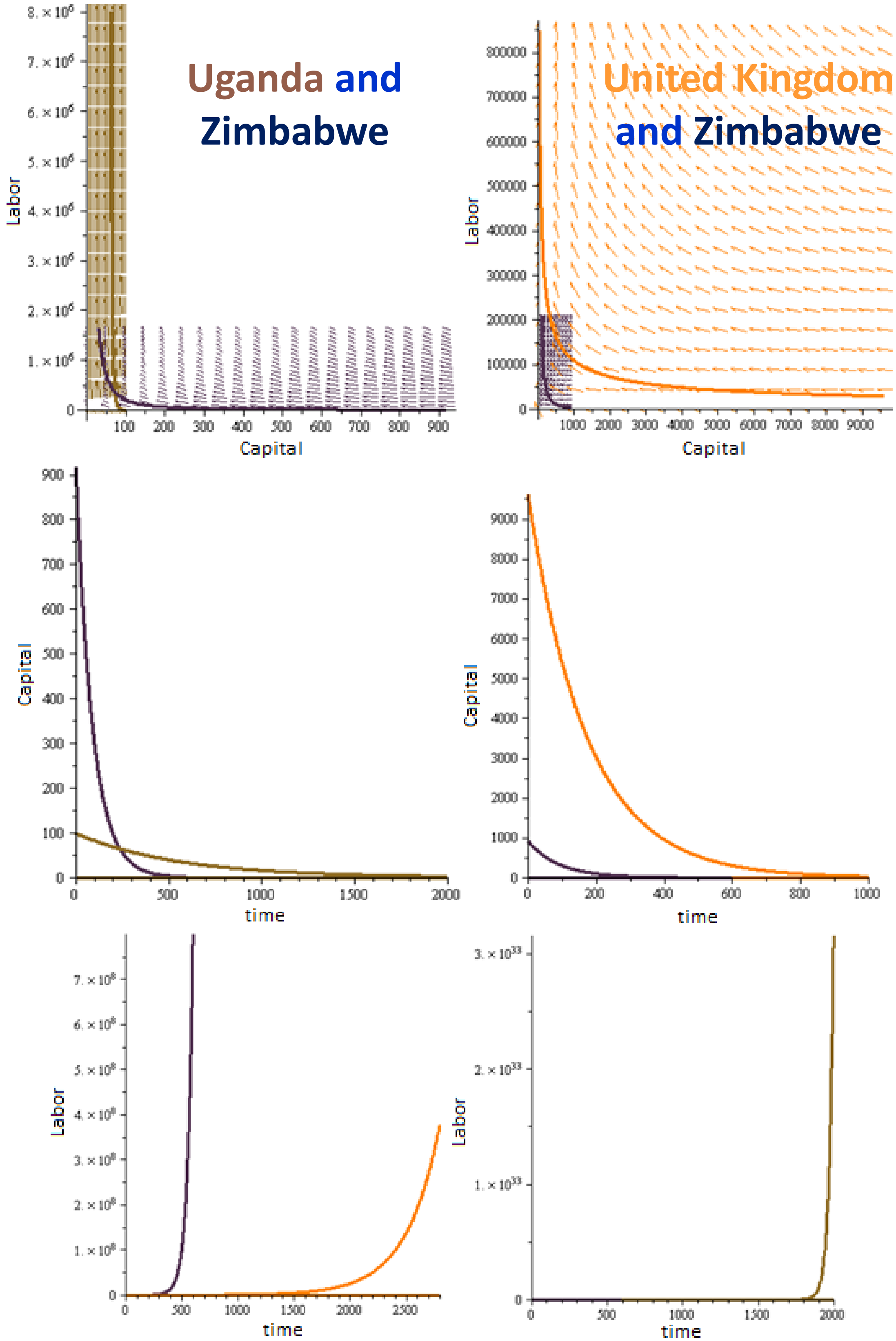
Spain
Capital
Time
United States
and
Uganda
Capital
Time
United States
and
United Kingdom
Capital
Time
United States
and
Zimbabwe
Capital
Time

Appendix D

$\alpha = 0$, s and d are constant , $Y = K(t)^0 * L^1 = L(t)$, $K' = s * L - dK(t)$



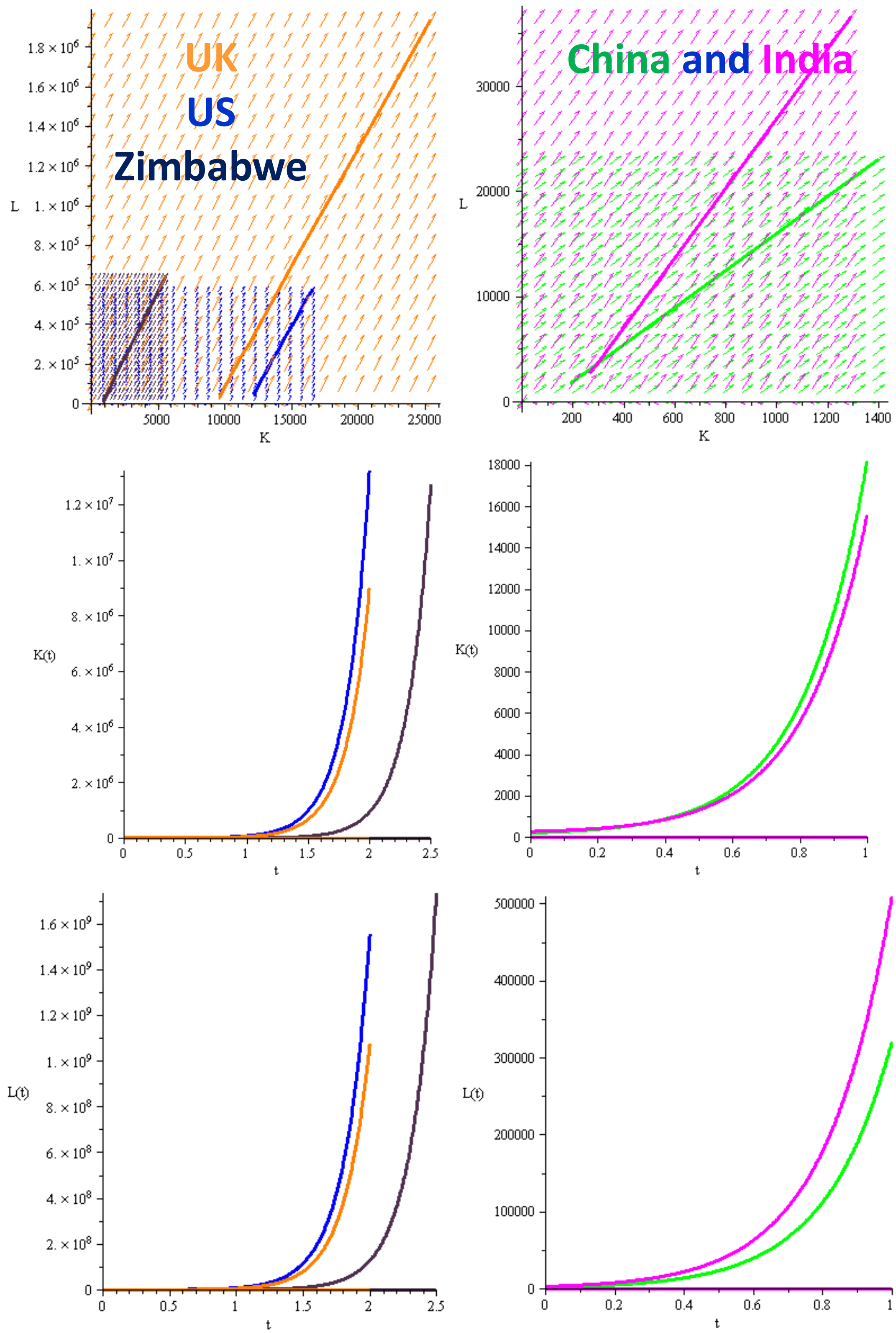

United States
and
United Kingdom
France
and Spain
Labor
Capital
time

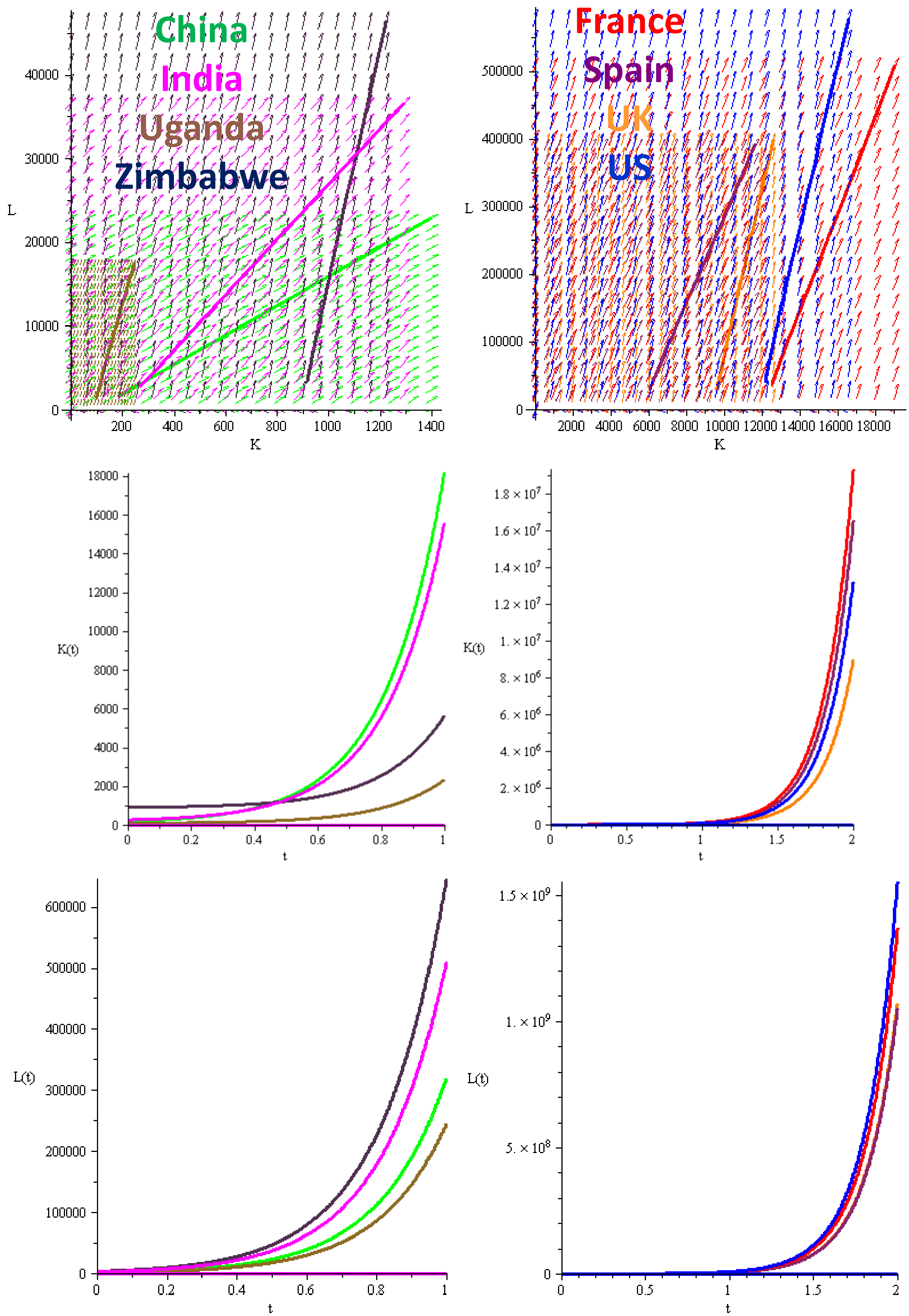


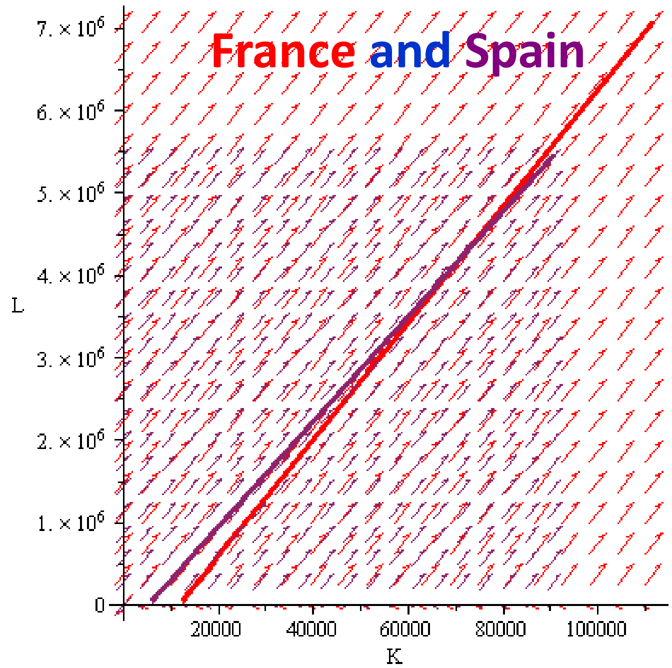
Uganda and Zimbabwe
United Kingdom and Zimbabwe
Labor
Capital
time

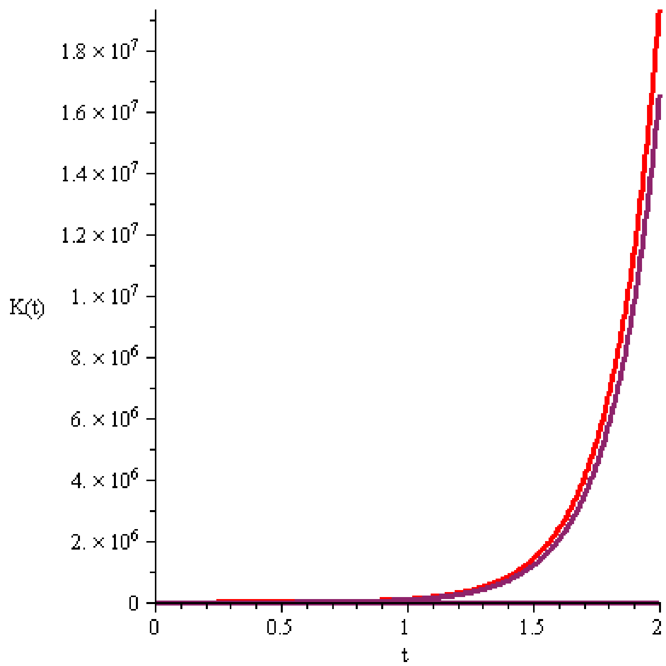
Appendix E

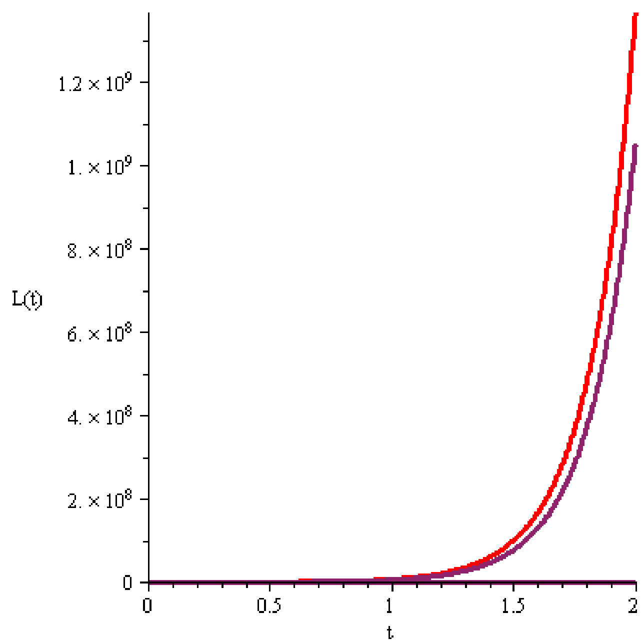
$\alpha = 1$, s and d are constant, $Y = K(t)^1 * L^0 = K(t)$, $K' = s * L - d * K(t)$, $L' = \frac{\alpha - \beta * L + \gamma * L^2}{\delta * L}$




China
India
Uganda
Zimbabwe
France
Spain
UK
US
L
K
K(t)
L(t)
t

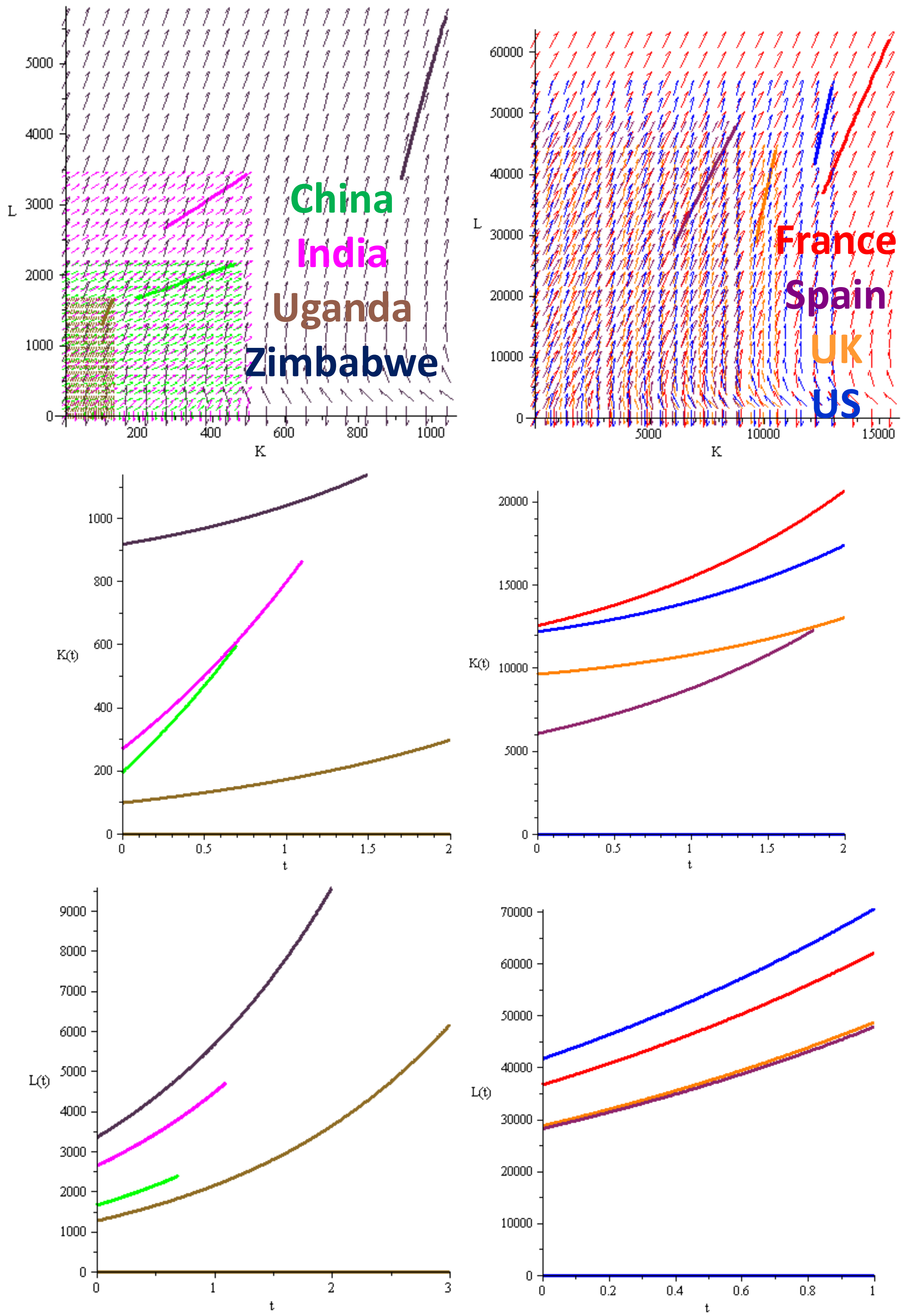

France and Spain
L
K
0
1. × 10^6
2. × 10^6
3. × 10^6
4. × 10^6
5. × 10^6
6. × 10^6
7. × 10^6
20000
40000
60000
80000
100000

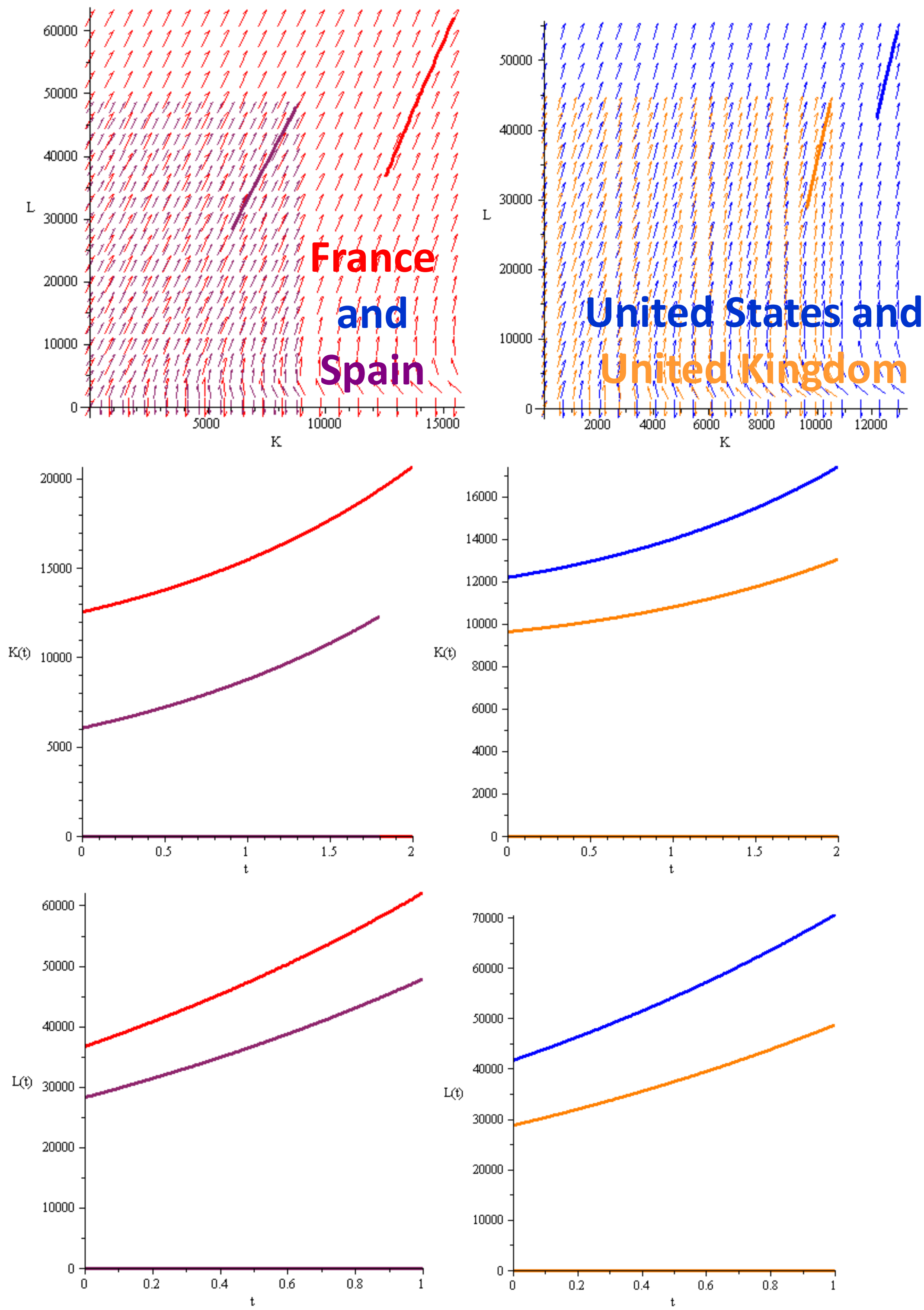

K(t)
t
0
2. × 10^6
4. × 10^6
6. × 10^6
8. × 10^6
1. × 10^7
1.2 × 10^7
1.4 × 10^7
1.6 × 10^7
1.8 × 10^7
0.5
1
1.5
2

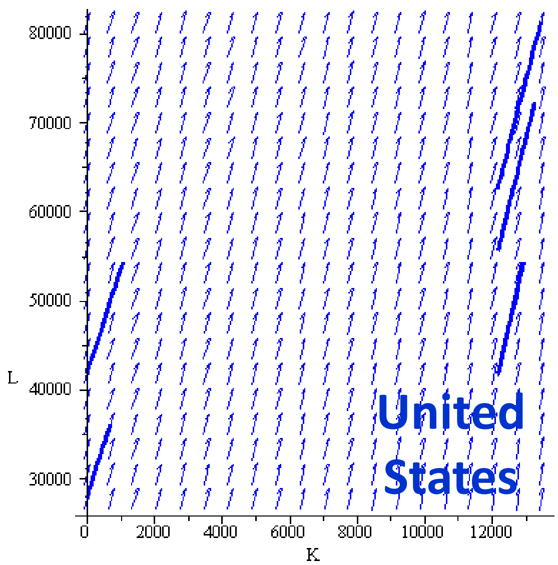

L(t)
t
0
2. × 10^8
4. × 10^8
6. × 10^8
8. × 10^8
1. × 10^9
1.2 × 10^9
0.5
1
1.5
2

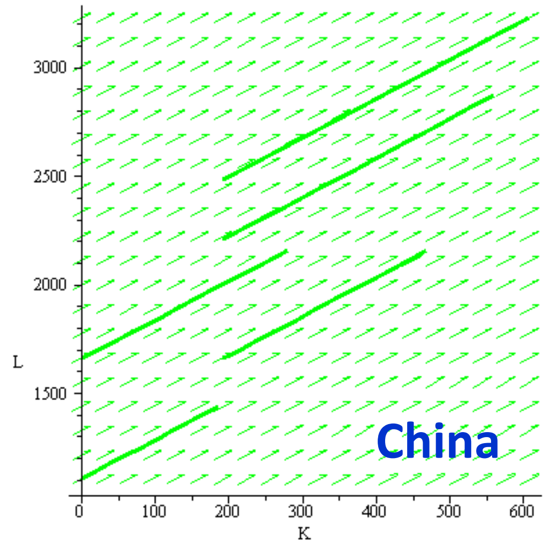
Appendix F

$\alpha = 1$, s and d constant, $Y = K(t)^1 * L^0 = K(t)$, $K' = s * L - d * K(t)$, $L' = \frac{\alpha - \beta * K + \gamma * L^2}{\delta * L}$




France
and
Spain
United States and
United Kingdom
L
K
K(t)
L(t)
t

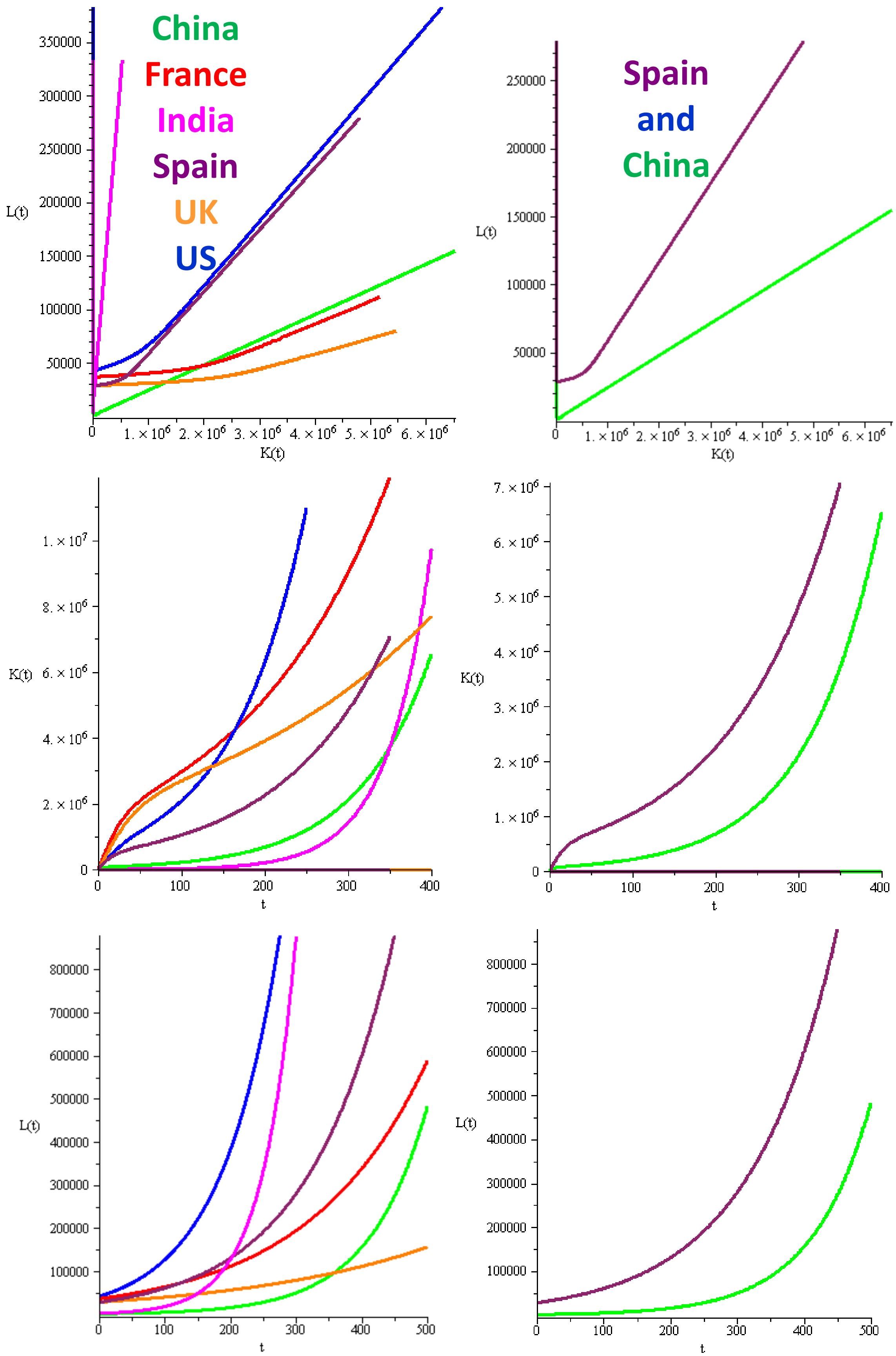


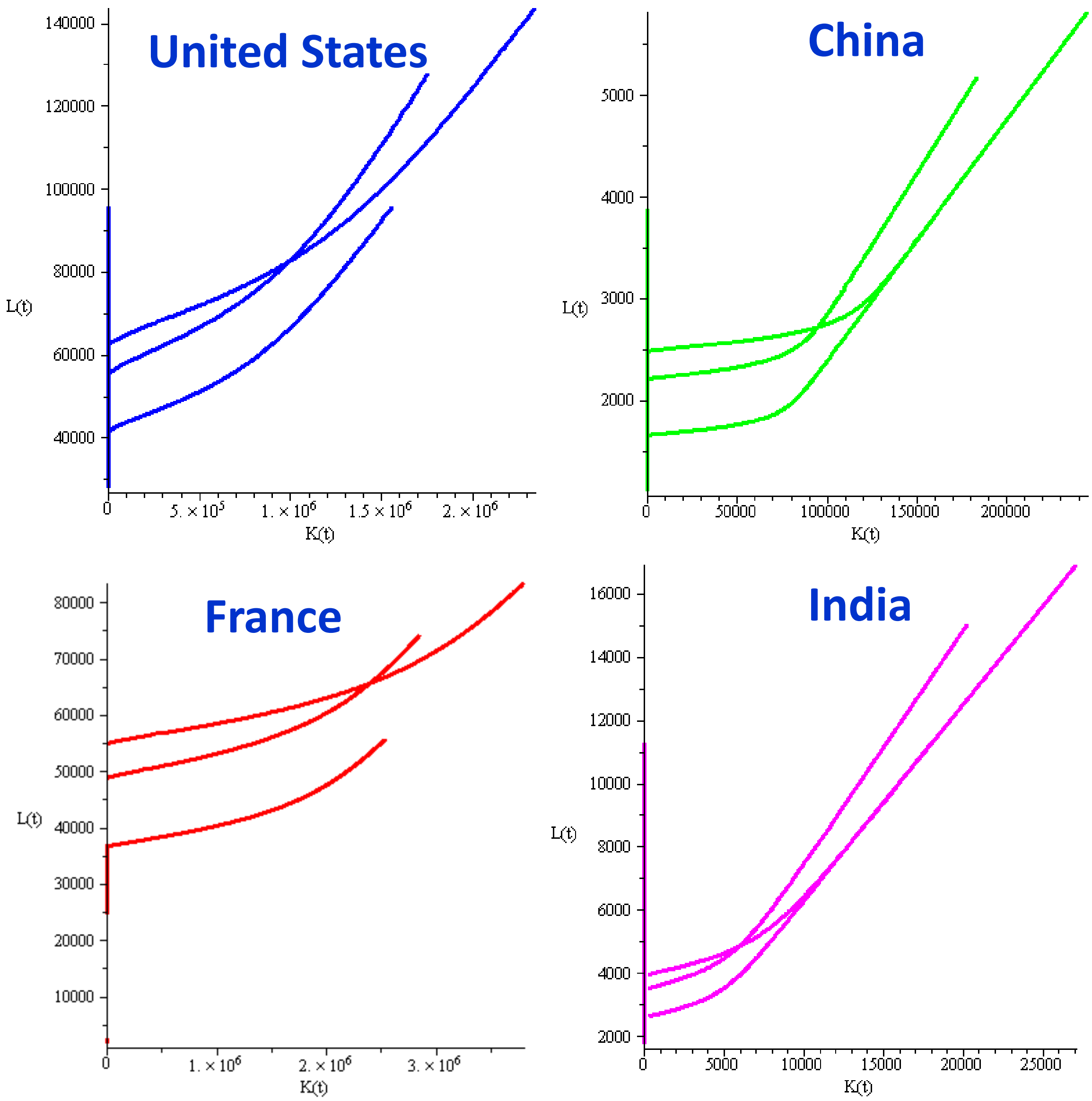


Appendix G

Add Technology and Education

Data Used					
	%pop in research	# of patent apps 1995	u for 1995	delta: growth of ideas	New savings rate
China	0.000448048	10011	6.11	0.264783954	0.352815052
France	0.002660095	12419	7.42	0.013471433	0.072165203
India	0.000153753	1545	4.52	0.121838529	0.192132215
Spain	0.001307363	2047	6.83	0.046424007	0.084193032
United Kingdom	0.002486819	18630	9.09	-0.00801482	0.05049382
United States	0.004178847	123962	11.89	0.051909542	0.034088464


China
France
India
Spain
UK
US
L(t)
K(t)
Spain
and
China
L(t)
K(t)
K(t)
t
K(t)
t
L(t)
t
L(t)
t


United States
140000
120000
100000
80000
60000
40000
L(t)
0
5. × 10^5
1. × 10^6
1.5 × 10^6
2. × 10^6
K(t)
China
5000
4000
3000
2000
L(t)
0
50000
100000
150000
200000
K(t)
France
80000
70000
60000
50000
40000
30000
20000
10000
L(t)
0
1. × 10^6
2. × 10^6
3. × 10^6
K(t)
India
16000
14000
12000
10000
8000
6000
4000
2000
L(t)
0
5000
10000
15000
20000
25000
K(t)