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Portfolio Diversification with Bitcoin

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Abstract

Bitcoin is a decentralized digital currency that was created in 2008. It has no central controlling authority, and allows payments to be sent from user to user without the need for a trusted third party or financial institution. When compared to traditional assets, Bitcoin exhibits persistently low correlations paired with exceedingly high volatility and returns. Using a modified mean-variance framework, we show that Bitcoin can be a viable diversification tool, but its investment appeal may be skewed by return activity that occured during a speculative bubble in 2013.

I. Introduction

Bitcoin is a decentralized digital currency that was created in 2008 by a pseudonymous programmer named Satoshi Nakamoto. Its peer-to-peer nature eliminates the need for a trusted third party, allowing for instantaneous payments to be sent anywhere in the world at essentially zero cost, and there is no central bank entity that controls the creation and distribution of Bitcoins. Instead, Bitcoins are created and secured through a process called mining. In order to "mint" new coins, Bitcoin miners run computer programs that compete to solve complex mathematical problems. The first miner to solve the problem is awarded one "block" of newly minted coins. In order to maintain a constant rate of block creation, the Bitcoin protocol automatically adjusts the difficulty of the mining problems, so that a new block is discovered every 10 minutes. The protocol also includes an algorithm that decreases the block reward by 50% every 210,000 blocks, or approximately every four years. This deflationary characteristic is designed to mimic the supply of traditional commodities like gold. Just as gold miners expend considerable resources in the form of labor and equipment, Bitcoin miners expend resources in the form of electricity and computing time in order to obtain a scarce (digital) commodity. (Nakamoto 2008)

Since its inception in 2008, Bitcoin adoption has grown at an astounding rate. According to blockchain.info, the number of unique Bitcoin wallet addresses has doubled in the last year alone. As of May 2016, one Bitcoin is worth approximately $500 and the total market capitalization of the Bitcoin network is over $8 billion. Bitcoin is actively traded on more than 60 online exchanges, and thousands of businesses – including Microsoft, Overstock.com and Dell – accept Bitcoins as payment for their products or services. Low transaction costs have helped it gain a foothold in the remittance market as well. According to a report published by the World Bank in 2015, the average remittance fee for sending $200 to 89 different countries is 5-8% of the transaction size. When compared to the average fee of $0.05 required to submit a Bitcoin transaction, its utility for remittances becomes clear. (World Bank Group 2015)

Along with its precipitous growth, Bitcoin has experienced several speculative bubbles, and the average daily volatility of the Bitcoin price is nearly 7 times higher than the S&P 500. There are also ongoing concerns about the ability of the Bitcoin network to handle large transaction volumes[1]. Because of these

[1] In its current state, the Bitcoin network cannot handle transaction volumes that compare to well established payment

issues, Bitcoin currently does not effectively operate as originally intended. High volatility impedes its ability to act as a store of value, while the network scalability issues hinder its viability as a medium of exchange.

Bitcoin has managed to disrupt several established domains, which has garnered considerable attention from academics in the fields of computer science and law. In spite of this, financial literature pertaining to Bitcoin is scarce. Although it is not currently a viable currency or long-term store of value, there is evidence that Bitcoin has merit as a digital asset. Briere, Oosterlinck, and Szafarz (2015) use spanning tests and a traditional mean-variance framework to show that including a small portion of Bitcoin in a well-diversified portfolio can substantially improve risk-return tradeoffs. More recently, Eisl, Gasser and Weinmayer (2015) apply a Conditional Value-at-Risk framework and portfolio backtesting techniques to yield similar results.

During 2013 and early 2014, daily log-returns to Bitcoin would regularly exceed 20% (with a max of 50%), causing risk/return ratios to be highly skewed when calculated over the entire sample. In order to account for this, we implement a "return penalty" paired with a modified mean-variance framework. Using this framework, we confirm the findings of Briere et al. (2015) and Eisl et al. (2015), then using backtesting techniques, we show that Bitcoin's ability to add value to an efficient portfolio may be overstated – largely due to the speculative bubble that occurred during 2013.

II. Data

Description

The first major Bitcoin exchange, Mt. Gox, was established in July 2010[2]. Due to the lack of consistent trading volume prior to 2012 (Figure 2), we omit data collected prior to January 1st, 2012. This leaves over 4 years of daily observations that span from January 2012 to May 2016. Historical daily Bitcoin prices are gathered from the CoinDesk Bitcoin Price Index, which is calculated as the midpoint of the bid/ask spread, and averaged across leading exchanges[3].

networks like Visa. (See https://en.bitcoin.it/wiki/Block_size_limit_controversy for more information on the block size limit.)

[2]Mt. Gox handled close to 70% of all Bitcoin trading volume by 2013. In early 2014, Mt. Gox suspended trading and announced that over $400 million in Bitcoin had been stolen. Mt. Gox filed for bankruptcy soon after. The founder of Mt. Gox was arrested and charged with embezzlement in August of 2015. (Popper 2014)

[3]The exchanges that meet the criteria to be included in the CoinDesk BPI are: Bitstamp, Bitfinex, Coinbase, itBit, and OKCoin. (See http://www.coindesk.com/price/bitcoin-price-index for information)

In order to represent the well-diversified portfolio of a U.S. investor, we use various index funds as proxies for US equities, developed foreign equities, real estate, commodities, and US bonds. For US equities, the SPDR S&P 500 ETF and iShares Russell 2000 ETF are used for large cap and small cap, respectively. Developed foreign equities are represented by the iShares EAFE ETF, real estate is represented by the Vanguard REIT ETF, commodities are represented by the iShares S&P GSCI Commodity-Indexed ETF, and U.S. investment grade bonds are represented by the Vanguard Bond Market ETF. All historical US ETF data are from Bloomberg. (Table 1)

Historical Performance and Return Characteristics

A performance comparison of Bitcoin and the assets in our representative portfolio is presented in Table 2. Regression coefficients and expected return calculations are obtained from the standard Capital Asset Pricing Model. (Sharpe 1964)

$$r_{i,t} - r_f = \alpha_i + \beta(r_{m,t} - r_f) + \epsilon_{i,t} \tag{1}$$

where

r_f is the risk free rate (we assume $r_f = 0$),

$r_{i,t}$ is the log-return to a particular stock or asset,

$r_{m,t}$ is the log-return to the market, and

β is the systematic risk associated with a given stock.

Table 2 also includes two risk adjusted performance metrics – the Sharpe Ratio (Sharpe 1994) and the Sortino Ratio. The Sharpe Ratio represents the risk-return tradeoff of holding a particular asset, with risk measured as the standard deviation of the asset returns. The Sortino Ratio is an adjustment to the Sharpe Ratio, and only considers the standard deviation of negative asset returns, or the downside deviation. The Sharpe Ratio is calculated,

$$Sharpe = \frac{\mu - r_f}{\sigma} \tag{2}$$

and the Sortino Ratio,

$$Sortino = \frac{\mu - r_f}{\sigma_d} \tag{3}$$

where

μ is the expected return,

σ is the standard deviation of returns, and

σ_d is the downside deviation of returns.

The CAPM fails to produce any significant β coefficients due to the higher moments of Bitcoin's return distribution. Extreme excess kurtosis and positive skewness indicate that a standard normal distribution is a poor approximation. Figure 3 shows the shape of the return distributions for BTC and SPY. This juxtaposition illustrates the unique characteristics of Bitcoin, as any number of factors could be the driving force behind this non-normal behavior (i.e. Bitcoin's age, its role as a currency or remittance vehicle, the health of the Bitcoin network itself, etc.). Nonetheless, Bitcoin does not easily easily fit into the traditional CAPM framework. Because of this, we will use the CAPM to estimate the expected returns of our other assets, and adopt a different method for approximating expected returns to Bitcoin.

Bitcoin's comparatively high volatility and low trading volume has left it susceptible to speculative bubbles in the past, and throughout the course of its turbulent history, major news sources and economic authorities have written Bitcoin's "obituary" on countless occasions.[4] Both of the significant bubbles–particularly the bubble spanning from 2013 to early 2014–further complicate our task of accurately measuring its expected return. In spite of this, we see that volatility continues to decrease. Figure 4 shows the GARCH(1,1) conditional volatility, with a one year moving average overlay. In the last 5 years alone, the average daily volatility of Bitcoin has decreased by over 60%, and (in terms of USD) the average daily trading volume has increased by a factor of 400.

Bitcoin also exhibits remarkably low correlation with every asset in our representative portfolio. Table

[4]As of May 21, 2015, Bitcoin has been referred to as "dead" or "dying" in 102 articles. ("Bitcoin Obituaries" 2015)

3 presents the Pearson correlation matrix. In order to examine the historical structure of the correlations, Figure 5 shows the same calculations plotted on a rolling 6 month window.

III. Methodology

An Adjusted Mean-Variance Framework

Since it is not highly correlated with any other traditional asset, and its CAPM β is not significantly different from zero, we hypothesize that Bitcoin can increase the performance of our portfolio by mitigating systematic risk. In order to evaluate the viability of Bitcoin as a part of an efficient portfolio – and to avoid imposing the complexity of Mean-VaR/C-VaR methods[5] – we use a traditional Markowitz mean-variance framework (Markowitz 1952), but with a slight adjustment. Because of the decreasing volatility and increasing trade volume, we believe that, as Bitcoin continues to grow, its return characteristics will also continue to stabilize. Instead of using the CAPM to calculate expected return for Bitcoin, we use the mean historical return with a magnitude-reducing "return penalty". Thus, our expected returns are given

$$\bar{r}_i = \begin{cases} \mu_i/\gamma, & \text{Bitcoin} \\ \beta_i \bar{r}_m, & \text{else} \end{cases} \tag{4}$$

where

$\bar{r}_i$ is the expected return to a given asset,

μ_i is the average historical return to a given asset,

γ is our imposed "return penalty",

β_i is the beta of a given asset, and

$\bar{r}_m$ is the Expected market return.

[5]The Mean C-VaR framework is designed to better account for non-normal distributions of asset returns. In spite of these differences, (Briere, Oosterlinck, and Szafarz 2015) and (Eisl, Gasser, and Weinmayer 2015) reach similar conclusions.

Mean-Variance Optimization

Let $\boldsymbol{w}$ represent a vector of security weights, $\boldsymbol{\Sigma}$ the covariance matrix of the security returns, and $\boldsymbol{R}$ a vector of expected returns. In order to form an efficient frontier of portfolios with optimal risk-return profiles, for a given "risk tolerance" q, we minimize[6]

$$\boldsymbol{w}^T \boldsymbol{\Sigma} \boldsymbol{w} - q \cdot \boldsymbol{R}^T \boldsymbol{w} \tag{5}$$

subject to constraints:

$$\sum_{i=1}^{N} w_i = 1$$

$$0 \leq w_i \leq p$$

where p is some maximum weight that any given asset can take.

Portfolio Compositions

In order to imitate the constraints of an average U.S. investor, we assume long only positions and no leverage. This assumes no preference about the weights, w_i our assets can take–only that we hold long positions.

$$0 \leq w_i \leq 1$$

Backtesting

In order to determine the efficacy of a portfolio containing Bitcoin, and in order to avoid lookahead bias, we implement a backtesting framework. Equal weights are assumed for the first three months of our data sample[7], then for the remaining portion, we rebalance on a quarterly basis using optimal weights (as determined by our mean-variance optimization). Once we obtain our two efficient portfolios–one with BTC and one without BTC–we backtest them over two different time periods in order to compare historical performance. The

[6] Minimization via "dual method". (Goldfarb and Idnani 1983)

[7] In order to make the most of our limited data sample, we assume the equal weight portfolio of (DeMiguel, Garlappi, and Uppal 2009) during the first three "training" months of our backtest.

first time period spans our entire data sample, while the second only includes returns after February 2014 (post-bubble).

IV. Results

The Efficient Frontier

Based on the real and risk adjusted performance of our portfolios (Table 4), we can conclude that portfolios containing Bitcoin tend to outperform their counterparts over our sample period. For our base case ($\gamma = 1$), the inclusion of BTC substantially increases the Sharpe Ratio from .98 to 1.57, and the Sortino Ratio from 1.42 to 2.17–a considerable increase in payoff per unit of risk. Allocating 14% of a well-diversified portfolio to BTC doubles annualized return (13% to 26%) with a comparatively small increase in risk (13% to 17%). Figure 6 shows the efficient frontier of optimal portfolios that include/exclude Bitcoin. Because BTC experiences such high levels of volatility, the minimum variance portfolio has no weight in Bitcoin, but the portfolio that maximizes the Sharpe Ratio contains a considerable weight of 14%. These results are consistent with the findings of Briere et. al (2015) and Eisl et al. (2015).

We also find that Bitcoin continues to add value at higher return penalty levels. For instance, if $\gamma = 10$ (i.e. the expected annual return of BTC is 10.6% instead of 106%), the tangency portfolio has nearly a 2% weight in BTC and a minimum 1% weight in BTC persists for return penalties up to $\gamma = 15$ (Figure 7). Figure 8 shows the efficient frontiers of several portfolios subject to various levels of our return penalty ($\gamma = 1, 2, 4, 8$).

Backtesting Portfolio Performance

Although BTC appears to astronomically improve the performance of a well-diversified portfolio, we assert that this out-performance is largely due to the speculative bubble of 2013. Figures 9 and 10 show the cumulative performance of portfolios including/excluding BTC; the first spanning the entire data sample, and the second spanning the period of returns that post-date the bubble. Here, we see that nearly all of the outperformance of the BTC portfolio stems from the bubble period, and if we compare post-bubble performance, the favorable risk/return tradeoffs of BTC disappear. Table 5 shows a comparison of annualized return, standard deviation, Sharpe, and Sortino ratios.

V. Conclusion

Using our adjusted mean-variance model, we have shown that Bitcoin appears to be an attractive investment that can substantially increase the return/risk ratios of an efficient portfolio–even when we impose considerable return penalties. These results align with the findings of Briere et al. (2015) and Eisl et al. (2015). Since this performance does not appear to persist into more recent years, we also posit that the majority of these positive results stem from the speculative bubble that Bitcoin experienced in 2013-2014. When data before February 2014 (the end of the bubble) is removed from our sample, portfolios containing BTC underperform their non-BTC counterparts–from both real and risk-adjusted standpoints. It is important, nonetheless, to acknowledge the short-comings of our methods. We find that the higher moments of Bitcoin's return distribution exhibit positive skewness and excess kurtosis, so some of the necessary assumptions for mean-variance portfolio optimization are violated. Furthermore, Bitcoin's young age translates into sparse data on price activity, which severely limits our ability to reach a verdict about its true return characteristics.

Theoretically, its value is a function of utility, but Bitcoin's various roles as a currency, a remittance vehicle, and a distributed consensus network (to name just a few) add a multitude of variables that can significantly affect its perceived utility – and consequently its price. Uncertainty about Bitcoin's true function provides an environment prone to speculative bubbles and violent price corrections. Provided that it does not ultimately fail, it is likely that Bitcoin will experience more periods of extreme volatility and returns, the magnitude of which, are unknown. With this in mind, we assert that Bitcoin's viability as a part of a well-diversified portfolio (in the near term) is contingent upon its ability to compensate "growing pain" volatility with high return and a continued low correlation. Our analysis conducted using the return penalty, γ, has also shown that the magnitude of Bitcoin's future price appreciation does not need to match the returns of 2013 and early 2014 in order to merit its inclusion in a well-diversified portfolio. In fact, penalties as high as $\gamma = 20$ still result in a small portion of an efficient portfolio being allocated to Bitcoin. Because of this, we assert that Bitcoin can still play a significant role in the well-diversified portfolio of a U.S. investor.

VI. Figures

Figure I.

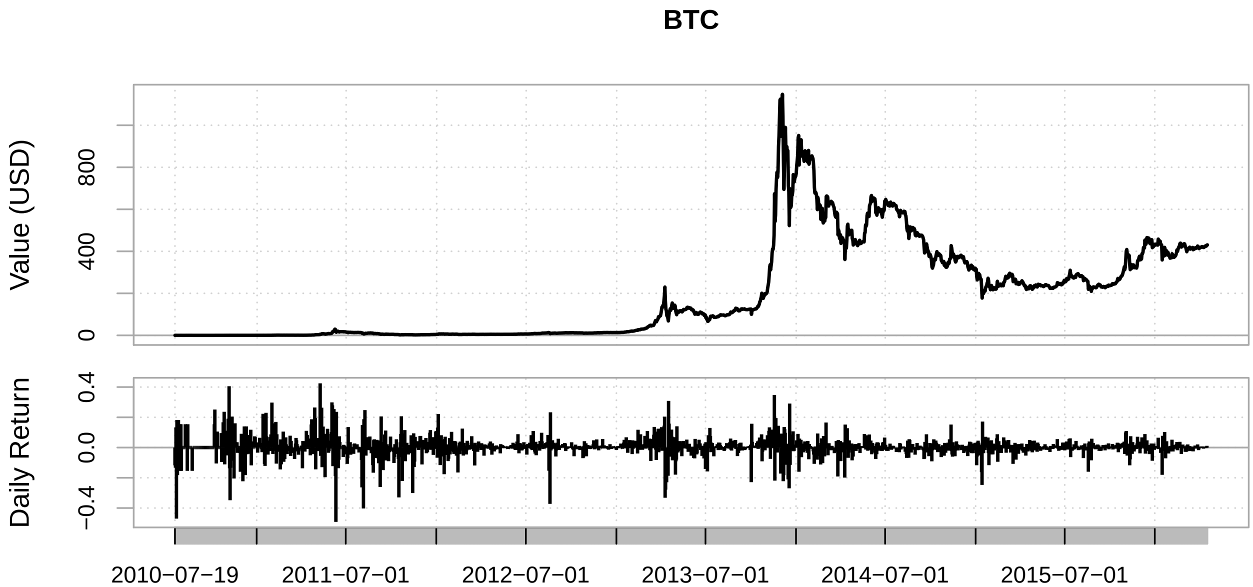

Figure I: Bitcoin price in USD (top) and daily log returns (bottom). Historical daily Bitcoin prices are gathered from the CoinDesk Bitcoin Price Index, which is calculated as the midpoint of the bid/ask spread, and averaged across leading exchanges.

Figure II.

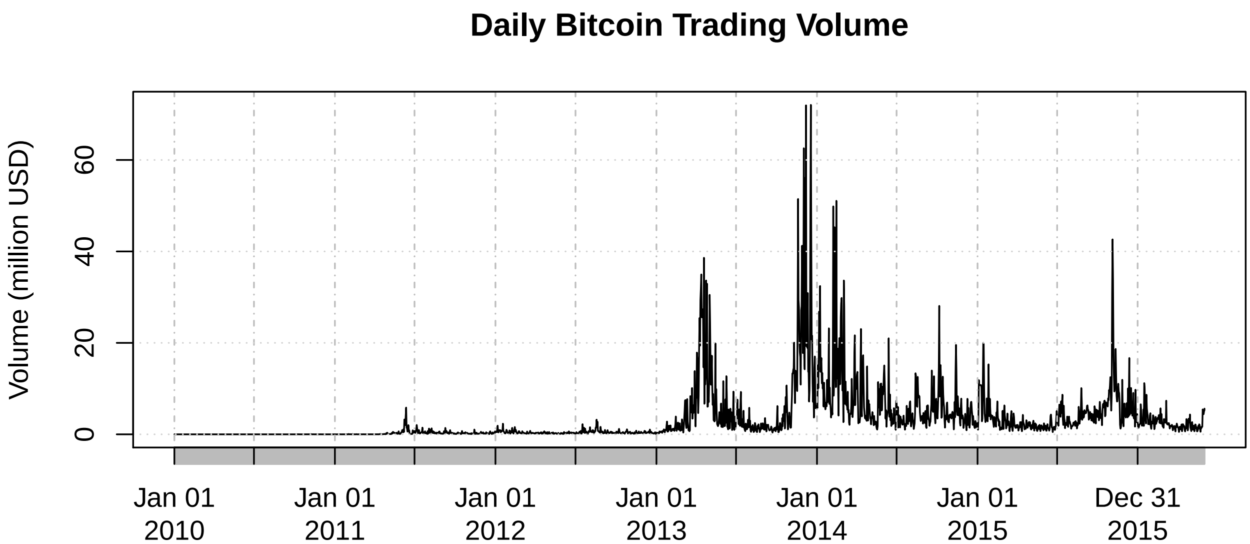

Figure II: Bitcoin trading volume in USD (million). Data from www.blockchain.info.

Figure III.

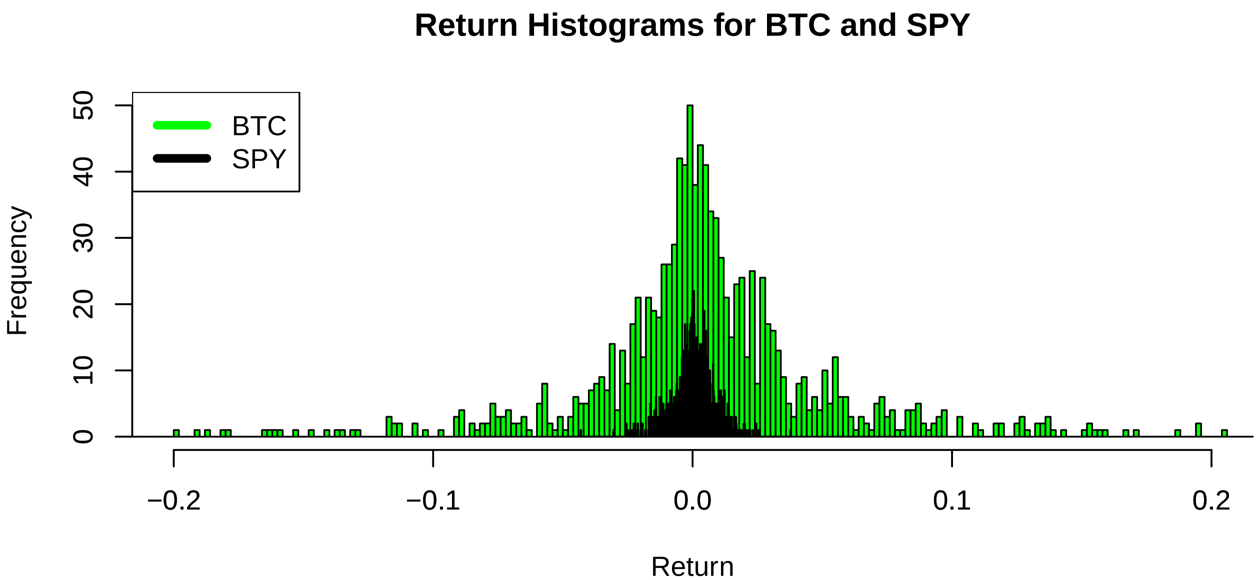

Figure III: Return histograms for Bitcoin and the S&P 500.

Figure IV.

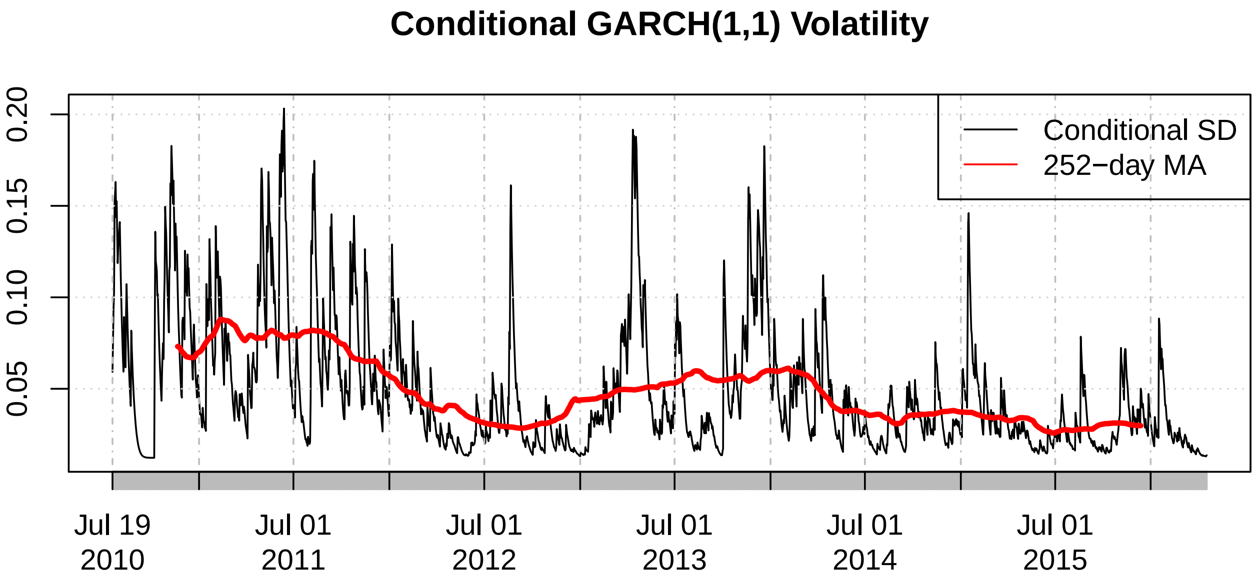

Figure IV: Conditional volatility of Bitcoin from 2010-2016. Fit using a GARCH(1,1) model and plotted with 252-day moving average overlay.

Figure V.

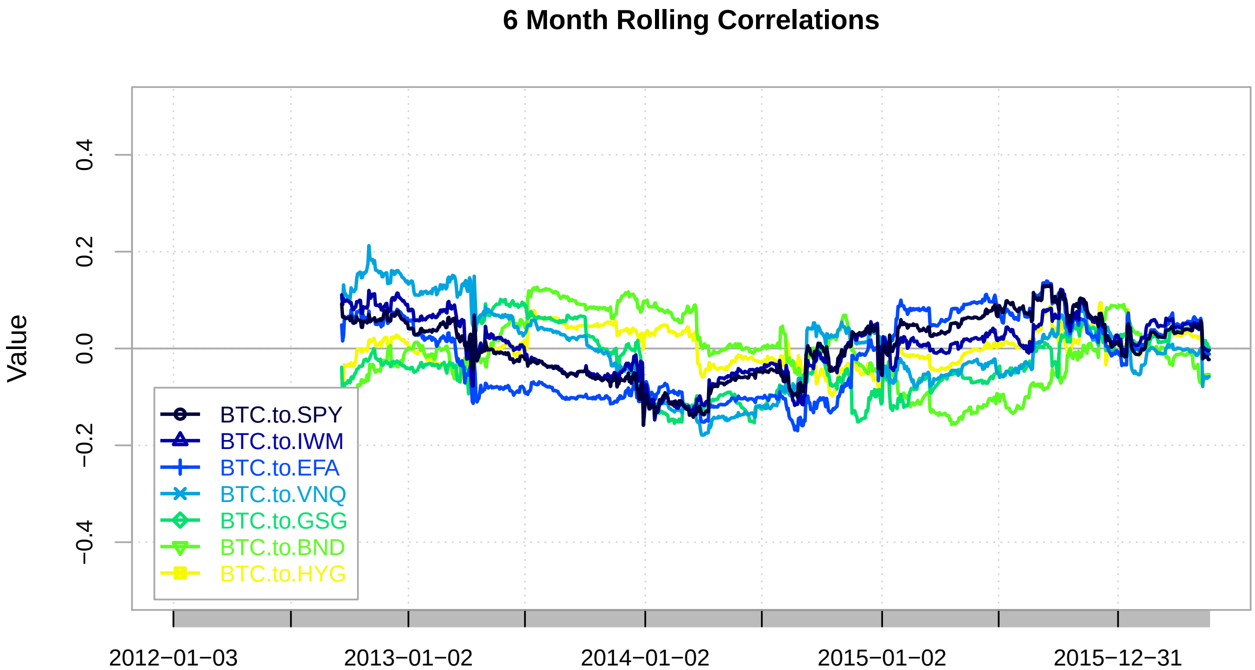

Figure V: Rolling correlations between Bitcoin and the other assets in our representative portfolio. Calculated using a rolling 6 month window.

Figure VI.

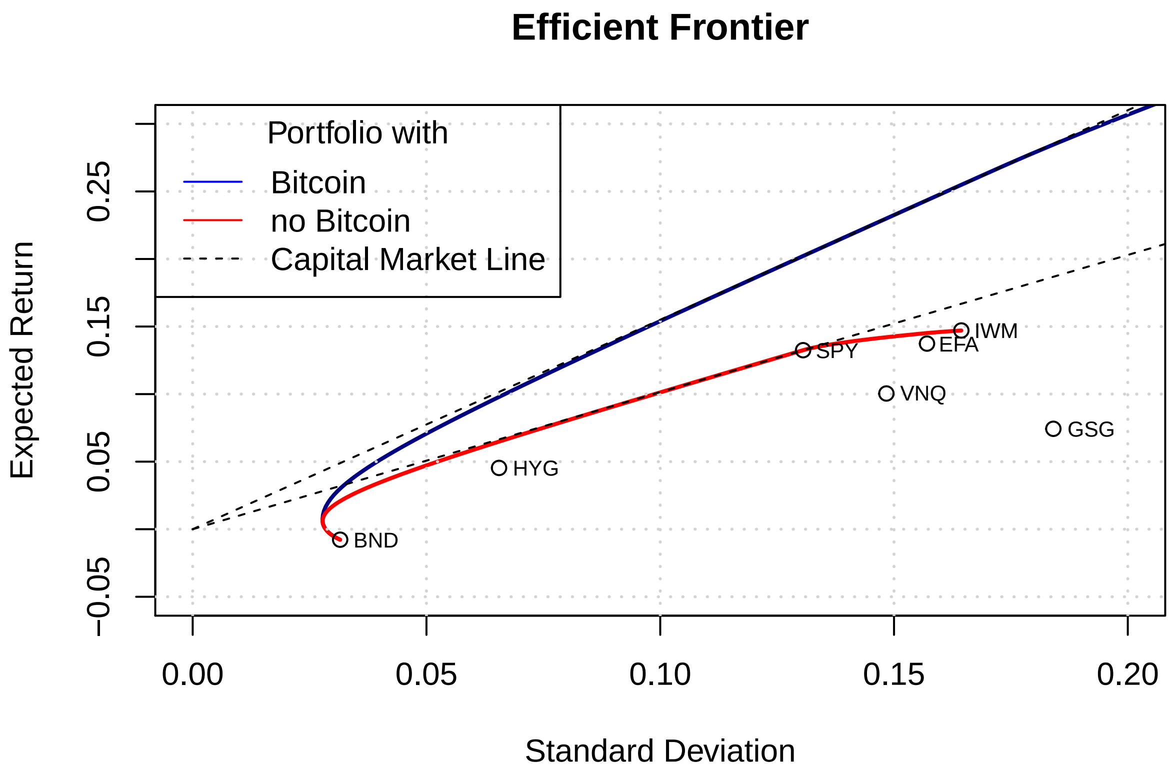

Figure VI: The efficient frontier of optimal portfolios including and excluding Bitcoin.

Figure VII.

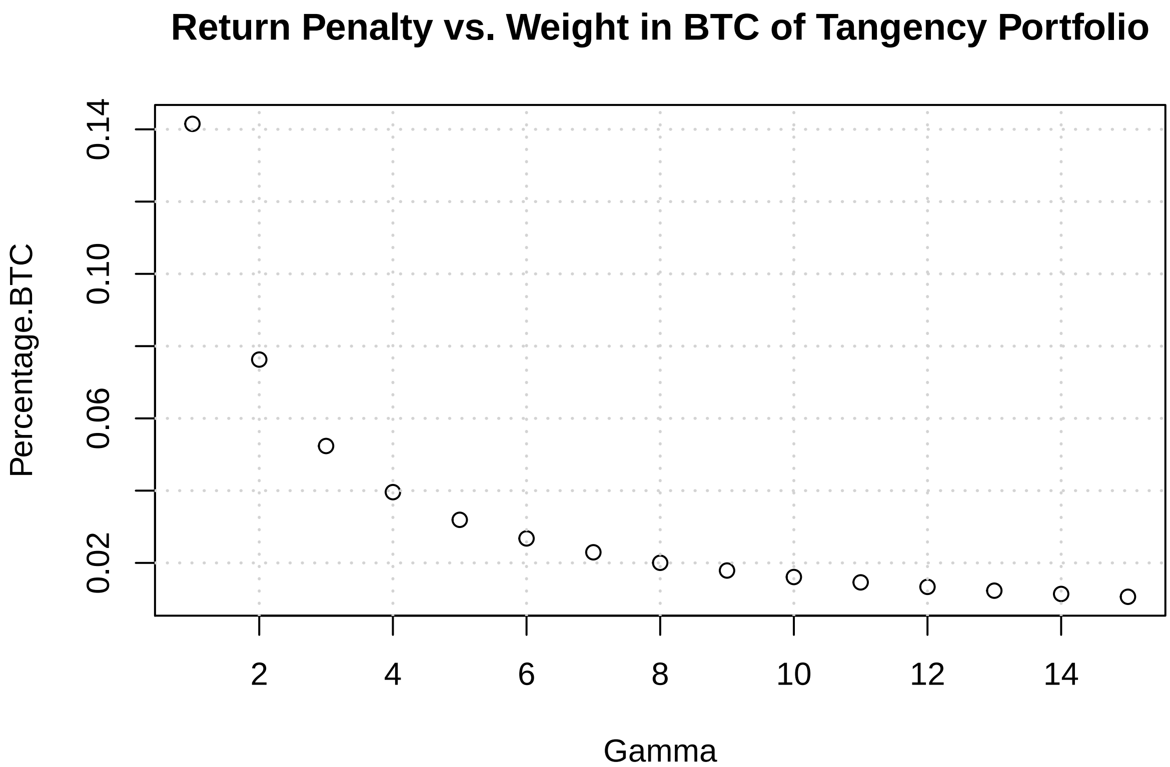

Figure VII: The percentage of an efficient max Sharpe portfolio allocated to BTC as a function of gamma.

Figure VIII.

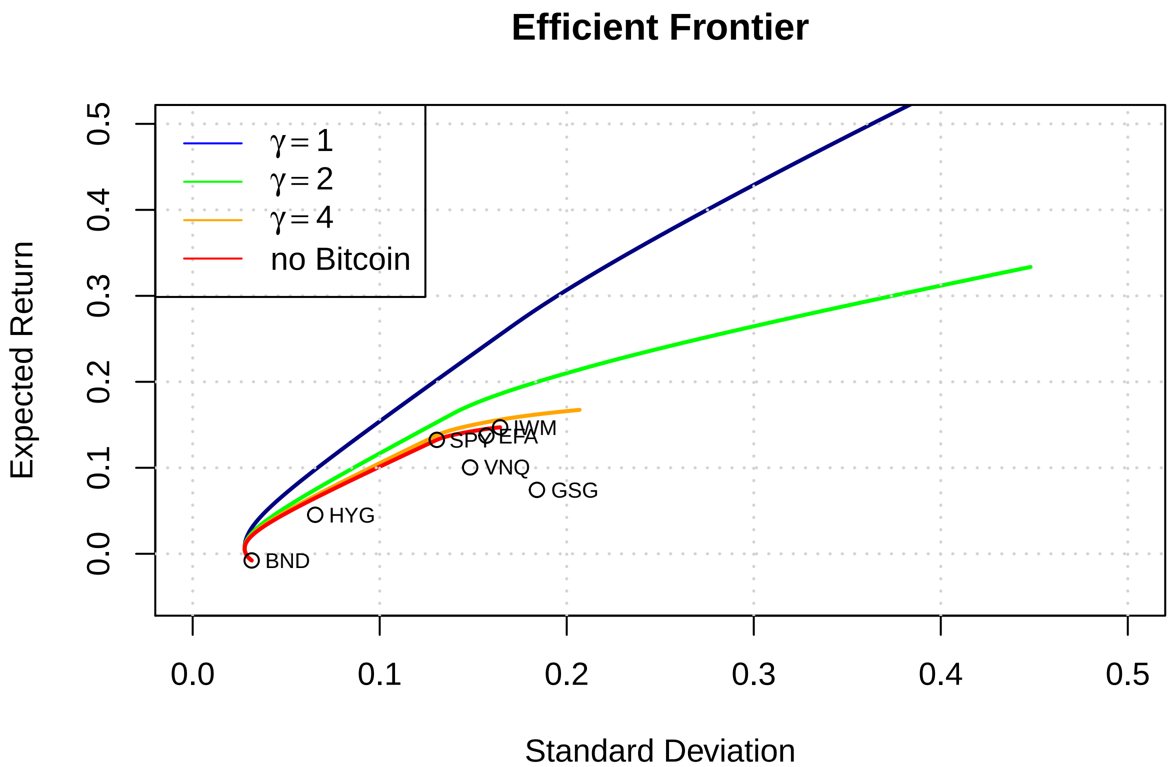

Figure VIII: Various efficient frontiers at different levels of our BTC return penalty, gamma. The maximum Sharpe portfolio contains at least a 1% weight in BTC for return penalties up to 15.

Figure IX.

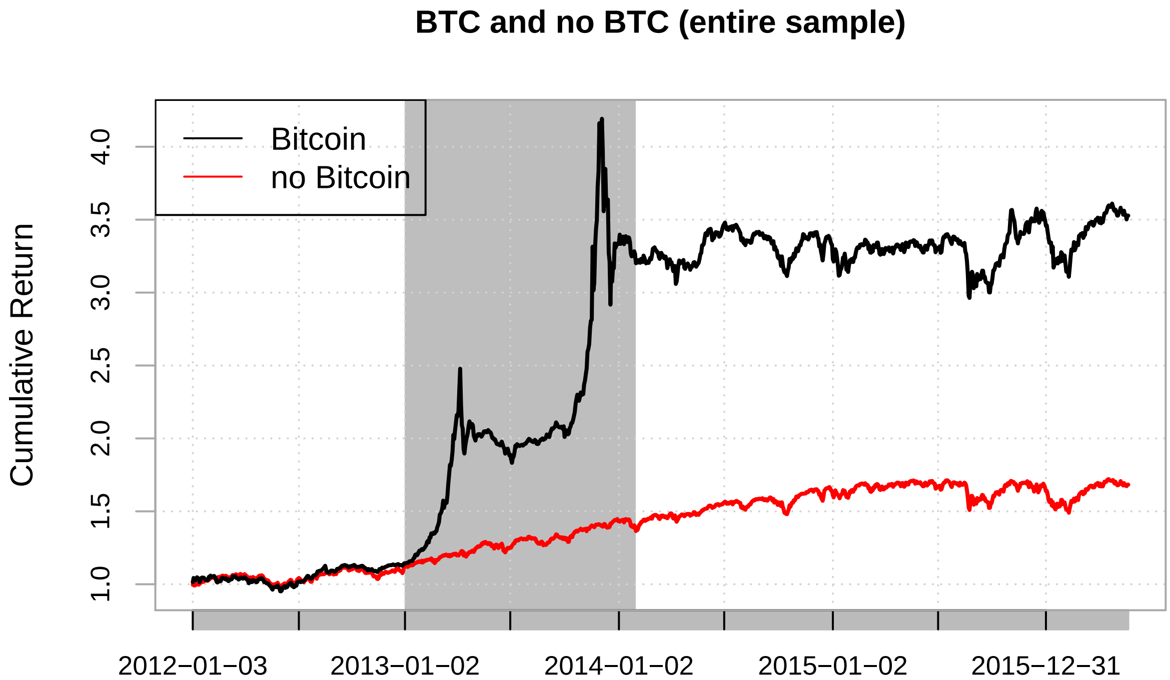

Figure IX: The cumulative return of portfolios that include/exclude BTC spanning the entire data sample. Portfolios are rebalanced on a quarterly basis using the tangency weights of a mean-variance optimization. The shaded region indicates the time period where the Bitcoin bubble took place.

Figure X.

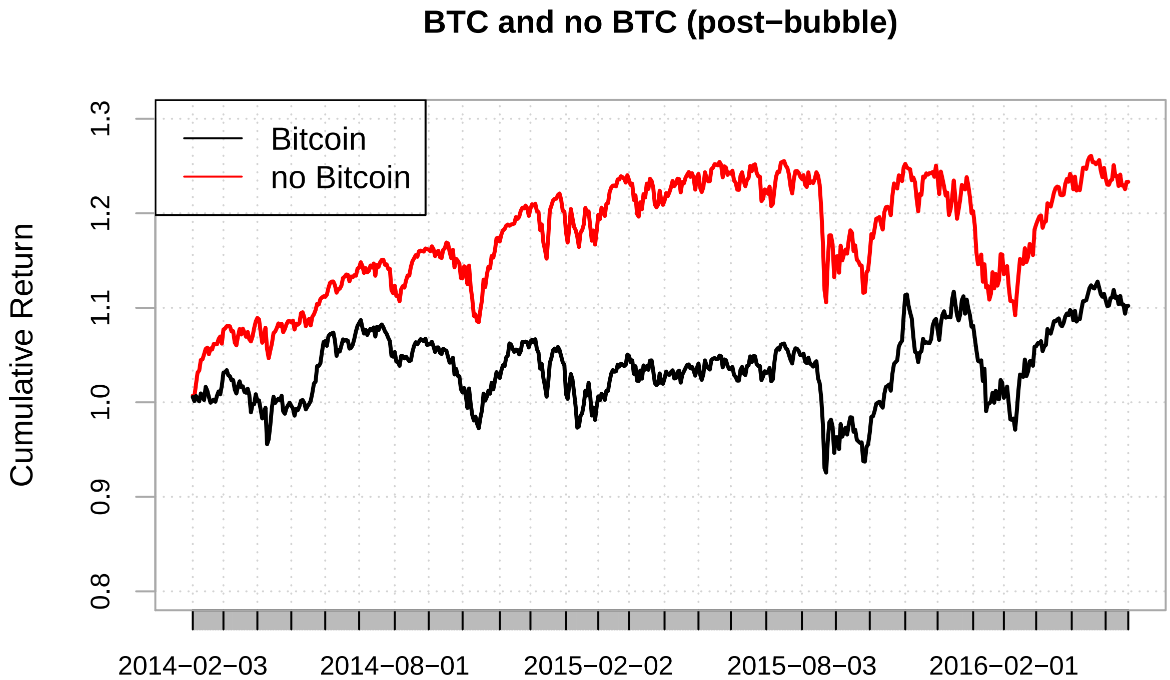

Figure X: The cumulative return of portfolios that include/exclude BTC spanning the last two years of the data sample (post-bubble). Portfolios are rebalanced on a quarterly basis using the tangency weights of a mean-variance optimization.

VII. Tables

Table 1: A Representative Portfolio

Asset Class	Representative Index	Ticker
Bitcoin	Coindesk BPI	BTC
Large Cap U.S. Equity	SPDR S&P 500 ETF	SPY
Small Cap U.S. Equity	iShares Russell 2000 ETF	IWM
Developed Foreign Markets	iShares MSCI EAFE ETF	EFA
Real Estate	Vanguard REIT ETF	VNQ
Commodities	iShares S&P GSCI Commodity-Indexed ETF	GSG
Bonds	Vanguard Total Bond Market ETF	BND
High Yield Bonds	iShares HY Bond ETF	HYG

Table 1: Each asset class, the accompanying index, and ticker. These will be used to form the representative "well-diversified" portfolio of a US investor.

Table 2: Historical Performance Metrics

	BTC	SPY	IWM	EFA	VNQ	GSG	BND	HYG
Avg. Annual Return	1.062	0.133	0.108	0.061	0.117	-0.174	0.023	0.041
CAPM Alpha	0.004	0.000	-0.000	-0.000	0.000	-0.001	0.000	0.000
CAPM Beta	0.017	1.000	1.110	1.037	0.758	0.561	-0.059	0.342
Correlation	0.002	1.000	0.882	0.862	0.667	0.398	-0.243	0.682
Correlation p-value	0.936	0.000	0.000	0.000	0.000	0.000	0.000	0.000
CAPM Expected Return	0.002	0.133	0.147	0.137	0.100	0.074	-0.008	0.045
Standard Deviation	0.905	0.131	0.164	0.157	0.148	0.184	0.032	0.066
Downside Deviation	0.611	0.091	0.117	0.112	0.106	0.136	0.023	0.045
Sharpe Ratio	0.003	1.015	0.895	0.875	0.677	0.404	-0.246	0.692
Sortino Ratio	0.004	1.457	1.259	1.226	0.945	0.547	-0.343	1.006
Skewness	0.125	-0.273	-0.270	-0.212	-0.459	0.028	-0.470	-0.012
Kurtosis	14.970	4.773	3.501	3.964	4.721	4.691	4.850	5.559

Table 2: Various performance/risk metrics and descriptive statistics, annualized.

Table 3: Pearson Correlation Coefficients

	BTC	SPY	IWM	EFA	VNQ	GSG	BND	HYG
BTC	1							
SPY	0.0024	1						
IWM	0.0046	0.8816	1					
EFA	-0.0163	0.8619	0.7687	1				
VNQ	-0.0118	0.6671	0.6246	0.5951	1			
GSG	-0.0156	0.3978	0.376	0.4428	0.1803	1		
BND	-0.0072	-0.2427	-0.2252	-0.1991	0.1157	-0.1278	1	
HYG	0.0021	0.6821	0.6297	0.6625	0.5264	0.4063	0.0036	1

Table 3: Pearson correlation matrix the assets in our representative portfolio, calculated over the entire data sample.

Table 4: Tangency Portfolio Comparison

	no BTC	$\boldsymbol{\gamma = 1}$	$\boldsymbol{\gamma = 2}$	$\boldsymbol{\gamma = 4}$	$\boldsymbol{\gamma = 8}$	$\boldsymbol{\gamma = 10}$
Ann. Return	0.13	0.26	0.20	0.17	0.15	0.14
Ann. Standard Dev	0.13	0.17	0.14	0.13	0.13	0.13
Sharpe Ratio	0.98	1.57	1.47	1.28	1.15	1.11
Sortino Ratio	1.42	2.17	2.04	1.80	1.63	1.59
Weight in BTC	0	0.14	0.08	0.04	0.020	0.016

Table 4: Annualized return, standard deviation, Sharpe Ratio, and Sortino Ratio of the tangency portfolios including/excluding BTC for various return penalty levels. (gamma greater than or equal to 1 implies BTC is included)

Table 5: Post-Bubble Backtest Results

	with BTC	without BTC
Ann. Return	0.03	0.08
Ann. Standard Dev	0.14	0.14
Sharpe Ratio	0.23	0.61
Sortino Ratio	0.41	0.93

Table 5: Annualized return, standard deviation, Sharpe Ratio, and Sortino Ratio of the portfolios which were backtested from 2014-2016.

VIII. References

"Bitcoin Obituaries." 2015. https://99bitcoins.com/bitcoinobituaries/.

Briere, Marie, Kim Oosterlinck, and Ariane Szafarz. 2015. "Virtual Currency, Tangible Return: Portfolio Diversification with Bitcoins." http://ssrn.com/abstract=2324780.

DeMiguel, Victor, Lorenzo Garlappi, and Raman Uppal. 2009. "Optimal Versus Naive Diversification: How Inefficient Is the 1/N Portfolio Strategy?" *The Review of Financial Studies* 22 (5): 1915–53.

Eisl, Alexander, Stephan Gasser, and Karl Weinmayer. 2015. "Caveat Emptor: Does Bitcoin Improve Portfolio Diversification?"

Goldfarb, D., and A. Idnani. 1983. "A Numerically Stable Dual Method for Solving Strictly Convex Quadratic Programs." *Mathematical Programming* 27 (August): 1–33.

Group, World Bank. 2015. *An Analysis of Trends in the Cost of Migrants Remittance Services*. World Bank Group.

Markowitz, Harry. 1952. "Portfolio Selection." *The Journal of Finance* 7 (1): 77–91.

Nakamoto, Satoshi. 2008. "Bitcoin: A Peer-to-Peer Electronic Cash System."

Popper, Nathaniel. 2014. "Apparent Theft at Mt. Gox Shakes Bitcoin World." The New York Times; Online. http://www.nytimes.com/2014/02/25/business/apparent-theft-at-mt-gox-shakes-bitcoin-world.html.

Sharpe, William F. 1964. "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk." *The Journal of Finance* 19 (3): 425–42.

———. 1994. "The Sharpe Ratio." *The Journal of Portfolio Management*.