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Revisiting the Financial Crisis: The Effect of Credit Shocks on Bond Yields

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From the financial crisis, it was apparent that traditional indicators such as real activity and inflation were insufficient to explain spikes in bond yields. I discover the effect of credit indicators on bond yields by estimating a Gaussian six-factor affine model of term structure. One of these factors is a credit variable that I construct using a principal component analysis of notable indicators. Using impulse response functions, I find that positive credit movements raise interest rates at all maturities. Furthermore, shocks to credit have a greater immediate impact compared to those of real activity which are milder and more persistent.

1 Introduction

I examine the impact of credit shocks on bond yields by estimating a Gaussian affine term structure model, using Ang and Piazzesi (2003). My contribution to the model is a credit factor that I construct using a principal component analysis of notable credit variables. After determining parameters for our model through a numerical optimization of a likelihood function, our model supports yield data for the past twenty years. To understand the impact of the credit factor, I implement impulse response functions on bond yields. I find that positive shocks to credit raise bond yields at all maturities of the yield curve. Because our credit variable is constructed such that positive shocks imply a looser credit environment, it is expected that positive impulses lower interest rates. In this way, our results contradict our expectations. Further, we find that credit shocks have an immediate impact on the yield curve while real activity has a milder and more persistent effect. We have some possible explanations for these findings which includes our particular construction of the credit variable and the validity of our large parametric maximum likelihood estimation.

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1.1 Motivation

In the aftermath of the most recent financial crisis, central banks were questioned in their approach to evaluating economic conditions. Beyond the traditional dual mandate of real growth and inflation, policymakers and lawmakers have called for a better management of asset prices and more specifically credit markets. In the United States, for example, there was a noticeable divergence between traditional economic indicators and measures of credit. Credit markets, measured through total consumer credit, experienced robust nominal growth rates, posting above 15% yearly growth in the mid 1990's and early 2000's. In Figures 1 and 2, we see that nominal growth rates of consumer and business credit have achieved large, absolute levels of fluctuation, in comparison to those of real output and inflation. Similarly, if we examine key private lending rates, including those of commercial paper and corporate bonds over the last two decades, fluctuations in key borrowing rates offset general stability in real economic activity and inflation. From this episode, is it fair to suggest that central banks should further utilize credit indicators in monetary policy? We will look to scratch the surface of this issue through the use of the yield curve.

Fundamentally, yield curves provide a large amount of information to economic agents. In particular, yields allow agents to gain some understanding of future expectations of inflation as well as prospects for economic growth. Because long term yields are the average of current and future short term interest rates, the yield curve is a device that can also be used to forecast. Another reason yield curves are so important is its relevance for asset pricing. By determining yields, alongside expectations of future cash flows, agents are able to estimate the price of any basic asset.

Yield curves are also extremely important to conducting monetary policy. Conventionally central banks directly affect the short end of the yield curve by adjusting short term interest rates. Ideally this translates into effects on the longer end and adjustments in investment and savings decisions. By understanding the factors that affect movements in yields, policymakers will gain further insight regarding ideal monetary policy and the determinants of economic conditions. In order to speak to the importance of credit indicators for policymaking, we will deal with a smaller and more manageable question - what is the effect of credit movements on bond yields?

To study this topic we extensively use Ang and Piazzesi. In its most basic form, this model utilizes a five-factor Gaussian vector autoregression (VAR) that incorporates three unobservable variables independent of two macroeconomic factors (real economic activity and inflation). By placing these factors side by side, it allows for easy comparison of both macro and latent effects on movements in yields. More specifically, this apparatus allows us to input disturbances with respect to different macroeconomic variables, similar to real life economic shocks. Our unique contribution to this model is an additional credit variable, which we construct through a number of data series. After estimating the model, we shock our credit variable, and see the resulting effect on the yield curve through the use of impulse response functions.

1.2 Affine Models

In an effort to better understand bond yields, affine or linear models of term structure characterize the yield curve as a linear function of the state variable. These models are especially useful because they are adapted (using certain assumptions on the state diffusion process) to allow for simple, closed form solutions of general term structure. Other term structure models that are characterized using non-affine structure deal with the main problem of stochastic and partial differential equations: they have no closed form solution and require costly numerical approximation, especially when dealing with higher dimensionality. Hence, discrete time affine models allow economists to gain tractability while providing a useful device to study bond yields. We will start by briefly discussing related literature in the field.

Vasicek (1977):
In this model, yields are expressed as a linear function of a single state variable, the short term interest rate. In discrete time, the state variable can be written as a first-order autoregression that is mean reverting (stochastically an Ornstein-Uhlenbeck diffusion process). The coefficient on the lag term controls the mean reverting speed of the process and can be calibrated to denote a random walk. Unfortunately the Vasicek model generates an average yield curve with less curvature than a similar curve constructed by data (Backus et al. (1998)). One of the key flaws of this model is that it allows for negative interest rates, which is infeasible in practice. The next model fixes this problem.

Cox, Ingersoll and Ross (CIR) (1985):
The CIR one-factor model is similar to the Vasicek model in that it also utilizes the short rate as its single state variable. In discrete time, the state variable is a function of both a first order lag term and a square root term. The major improvement of this model is that CIR guarantees nonnegative interest rates, through the use of this additional term. As the short rate approaches zero, it removes the effect of any error, and emphasizes the positive drift term. Its other results however are analogous to those of the previous model, as the average model-generated yield curve exhibits less curvature than in reality. The main deficiency of both these models is the inability of one factor to sufficiently explain data due to a limited parametrization.

Chen and Scott (1993):
This model is a multifactor expansion of CIR, with separate models for 1, 2, and 3 unobservable variables. This paper is particularly unique in its utilization of maximum likelihood estimation to find parameters. Furthermore the paper introduces the concept of yield measurement errors as there are more observable yields than factors. We briefly introduce this paper here as it will be useful for our own model estimation later on.

Duffie and Kan (1996):
This groundbreaking paper was the first to classify a generalized multifactor version of affine models. It accounts for multidimensional expansions of Vasicek (as done by Langetieg (1980)), CIR (which was included in the original paper), as well as Chen and Scott. Its main improvement however, was its ability to account for latent or unobservable variables. Given observable yields, we can specify unobservable factors and back them out. This is similar to work by Litterman and Schienkman (1988) in which they identify three unobservable factors as "level", "slope", and "curvature." The model we will be using is a version of Duffie and Kan in which we incorporate both observable and unobservable factors.

1.3 Basic Bond Math

Before we continue, it is important that we establish conventions for bond pricing. In particular $yields$ can be characterized through $prices$ of bonds. For zero coupon securities, we know that the payoff is some fixed amount at maturity. We will assume this value to be unitary. This implies that the gross n-period return, $R = \frac{1}{P_t^n}$, where P_t^n is a price of an n-period bond at time t. Using fact that log(1+r) $\approx$ r, for small values of r,

$$\log(R) = \log(1) - \log(P_t^n) = -\log(P_t^n)$$

is our net return. Furthermore, the per-period "yield to maturity" is equivalent to:

$$y_t^n = \frac{\log R}{n} = \frac{-\log(P_t^n)}{n} \tag{1}$$

Next we use the central asset pricing equation, derived from the consumer portfolio choice problem:

$$E_t(m_{t+1}R_{t+1}) = 1 \tag{2}$$

where, E_t is the expected value operator conditional on information available at time t. m_{t+1} is a random variable that we will refer to as the "stochastic discount factor" or "pricing kernel." R_{t+1} is the gross return from period t to period $t+1$.

Suppose we are considering an $n+1$ period zero coupon bond at time t (and accordingly an n period zero coupon bond at time $t+1$). We can rewrite this gross return at period $t+1$ as:

$$R_{t+1} = \frac{P_{t+1}^n}{P_t^{n+1}}$$

where, P_{t+1}^n is the price of an n-period bond at time $t+1$. We plug this expression into (2) and because P_t^{n+1} is a known quantity at time period t, by manipulation we have:

$$E_t(m_{t+1}P_{t+1}^n) = P_t^{n+1} \tag{3}$$

We will use these facts in the next section, to derive the yield curve for our affine model.

2 Model

In this section we will walk through the model that is originally specified by Ang and Piazzesi. For enrichment purposes, we will go through some of the major derivations and construct the main apparatus that will be used.

2.1 State Variable

Our affine model will revolve around a state variable, X_t, which denotes an $n \times 1$ vector at time period t. The state variable will be composed of two elements: an observable part (that will be derived from macroeconomic data) and an unobservable portion that we will look to back out through estimation. Used in previous literature, these unobservable variables play a major role in fluctations of the yield curve.

Similar to Litterman and Schienkman, among others, we will utilize three factors to incorporate the unobservables. For the remaining $n-3$ observable factors, we will use data related to real economic activity and inflation (similar to original paper) and add a credit variable that will be constructed using principal components of common credit indicators. The law of motion for the state variable will be as follows:

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t \tag{4}$$

where μ is an $n \times 1$ vector, Φ and Σ are $n \times n$ matrices. ε_{t+1} is an $n \times 1$ Gaussian random vector of shocks. Note that (4) is a first order vector autoregression (VAR) of X_t.

2.2 Short Rate

As a strict assumption of all affine models, the short rate dynamics will be specified as a linear function of the state.

$$r_t = \delta_0 + \delta_1' X_t \tag{5}$$

where r_t is the short rate, δ_0 is a scalar, δ_1' is the transpose of an $n \times 1$ vector.

2.3 Market Price of Risk

The market price of risk measures the additional return needed in exchange for increased uncertainty. In this model the market price of risk, λ_t, will also be a linear function of the state.

$$\lambda_t = \lambda_0 + \lambda_1 X_t, \tag{6}$$

where λ_0 is an $n \times 1$ vector and λ_1 is $n \times n$ matrix.

2.4 Stochastic Discount Factor

We will designate a functional form of the stochastic discount factor, m_t, which will allow us to backout a recursion for the yield curve.

$$\log(m_{t+1}) = -\frac{1}{2}\lambda_t'\lambda_t - r_t - \lambda_t'\varepsilon_{t+1}, \tag{7}$$

where ε_{t+1} is an $n \times 1$ vector of shocks. Using equations (7) and (5) we have the following result:

$$\log(m_{t+1}) = -\frac{1}{2}(\lambda_0' + X_t'\lambda_1')(\lambda_0 + \lambda_1 X_t) - (\delta_0 + \delta_1' X_t) - (\lambda_0' + X_t'\lambda_1')\varepsilon_{t+1}$$

$$m_{t+1} = \exp(-\frac{1}{2}(\lambda_0' + X_t'\lambda_1')(\lambda_0 + \lambda_1 X_t) - (\delta_0 + \delta_1' X_t) - (\lambda_0' + X_t'\lambda_1')\varepsilon_{t+1}).$$

2.5 Yield Curve

Suppose that yields are affine functions of the state. This means that we can describe our yields in the following form:

$$-\log P_t^n = A_n + B_n' X_t, \tag{8}$$

where $-\log P_t^n$ is our multiple-period yield and A_n and B_n are functions of n, the number of periods till maturity.

Using (3) and (8) we can rewrite our bond prices as the following:

$$\begin{aligned} P_t^{n+1} &= E_t(m_{t+1} P_{t+1}^n) \\ &= E_t(\exp(-\frac{1}{2}(\lambda_0' + X_t'\lambda_1')(\lambda_0 + \lambda_1 X_t) \\ &\quad - (\delta_0 + \delta_1' X_t) - (\lambda_0' + X_t'\lambda_1')\varepsilon_{t+1}) P_{t+1}^n) \end{aligned}$$

We know for a one-period bond (n=1) the payoff in the last period is 1. This allows to rewrite the preceeding equation as:

$$P_t^1 = E_t(\exp(-\frac{1}{2}(\lambda_0' + X_t'\lambda_1')(\lambda_0 + \lambda_1 X_t) - (\delta_0 + \delta_1' X_t) - (\lambda_0' + X_t'\lambda_1')\varepsilon_{t+1}))$$

Notice here that the contents of the exponential are a normal random variable. We know for a random variable $Y = \exp(X)$, where X is Gaussian, Y follows a log normal distribution whose expectation, $E(Y) = \exp(\mu + \frac{\sigma^2}{2})$, where μ and σ^2 are the mean and variance of X. Equivalently, $\log(E(\exp(X))) = \mu + \frac{1}{2}\sigma^2$. Using this important fact we then have that:

$$\begin{aligned} -\log P_t^1 &= \frac{1}{2}(\lambda_0' + X_t'\lambda_1')(\lambda_0 + \lambda_1 X_t) + (\delta_0 + \delta_1' X_t) \\ &\quad - \frac{1}{2}(\lambda_0' + X_t'\lambda_1')(\lambda_0 + \lambda_1 X_t) \\ &= \delta_0 + \delta_1' X_t \end{aligned}$$

This results from the fact that $\varepsilon_{t+1} \sim N(0, I)$. If we put this side by side equation (8), we find that $A_1 = \delta_0$ and $B_1' = \delta_1'$.

Using the methodology outlined by Backus et al., among others, we will use induction to find a recursive function for A_{n+1} and B_{n+1}. From equation (3) we have:

$$\begin{aligned} P_t^{n+1} &= E_t(m_{t+1} P_{t+1}^n) = E_t(m_{t+1} \exp(-A_n - B_n X_{t+1})) \\ &= E_t(m_{t+1}(\exp(-A_n - B_n X_{t+1}))) \\ &= E_t(m_{t+1}(\exp(-A_n - B_n'(\mu + \Phi X_t + \Sigma\varepsilon_{t+1})))) \\ &= E_t(\exp(-\frac{1}{2}(\lambda_0' + X_t'\lambda_1')(\lambda_0 + \lambda_1 X_t) - (\delta_0 + \delta_1' X_t) \\ &\quad - (\lambda_0' + X_t'\lambda_1')\varepsilon_{t+1} - A_n - B_n'(\mu + \Phi X_t + \Sigma\varepsilon_{t+1})). \end{aligned}$$

Let Y be the contents of the exponential:

$$\mu_Y = -\frac{1}{2}(\lambda_0' + X_t'\lambda_1')(\lambda_0 + \lambda_1 X_t) - (\delta_0 + \delta_1' X_t) - A_n - B_n'\mu - B_n'\Phi X_t$$

$$\begin{aligned}\sigma_Y^2 &= E_t((-\lambda_0' - X_t'\lambda_1' - B_n'\Sigma)\varepsilon_{t+1}\varepsilon_{t+1}'(-\lambda_0 - \lambda_1 X_t - \Sigma' B_n)) \\ &= (\lambda_0' + X_t'\lambda_1' + B_n'\Sigma)(\lambda_0 + \lambda_1 X_t + \Sigma' B_n) \\ &= (\lambda_0' + X_t'\lambda_1')(\lambda_0 + \lambda_1 X_t + \Sigma' B_n) + B_n'\Sigma(\lambda_0 + \lambda_1 X_t + \Sigma' B_n) \\ &= (\lambda_0' + X_t'\lambda_1')(\lambda_0 + \lambda_1 X_t) + 2B_n'\Sigma(\lambda_0 + \lambda_1 X_t) + B_n'\Sigma\Sigma' B_n.\end{aligned}$$

Using the properties of the log normal distribution as before,

$$\begin{aligned}-\log P_t^{n+1} &= -\mu_Y - \frac{1}{2}\sigma_Y^2 \\ &= \delta_0 + \delta_1' X_t + A_n + B_n'\mu + B_n'\Phi X_t - B_n'\Sigma(\lambda_0 + \lambda_1 X_t) - \frac{1}{2}B_n'\Sigma\Sigma' B_n \\ &= \underbrace{(A_n + B_n'(\mu - \Sigma\lambda_0) - \frac{1}{2}B_n'\Sigma\Sigma' B_n + \delta_0)}_{\mathbf{A_{n+1}}} + \underbrace{(B_n'(\Phi - \Sigma\lambda_1) + \delta_1')}_{\mathbf{B'_{n+1}}} X_t.\end{aligned}$$

Hence, the coefficients of the affine yield model can be expressed recursively as:

$$A_{n+1} = A_n + B_n'(\mu - \Sigma\lambda_0) - \frac{1}{2}B_n'\Sigma\Sigma' B_n + \delta_0, \tag{9}$$

$$B_{n+1}' = B_n'(\Phi - \Sigma\lambda_1) + \delta_1', \tag{10}$$

where $A_1 = \delta_0$ and $B_1' = \delta_1'$. Furthermore, we specify the per period "continuous" yield as:

$$-\frac{\log P_t^n}{n} = \frac{A_n}{n} + \frac{B_n}{n} X_t \tag{11}$$

These results agree with the model derived originally by Ang and Piazzesi. Using this structure, we will estimate parameters and build yield curves through the use of macroeconomic factors.

3 Estimation

In this section we will go through the "two-step" routine used by Ang and Piazzesi to estimate unknown parameters. First, we will do preliminary estimations of observable variables and the short rate equation. Along the way we will place some restrictions on our parameters to make the estimation much simpler, as we eliminate a number of free parameters. Towards the end we will specify the likelihood function, which assigns a joint probability of bond yields and state variables, given a set of parameters. To solve our model, we seek to choose parameters that maximize this likelihood function.

3.1 Preliminary Steps

Our complete model will contain a 6×1 state variable ($n = 6$) with three observable and three unobservable factors. Following the parametrizations used by Ang and Piazzesi, we assume that the observable factors take form in the following system:

$$X_t^o = \rho^o X_{t-1}^o + \Omega u_t^o \tag{12}$$

where X_t^o is the 3×1 observable portion of X_t, ρ^o is a 3×3 matrix and Ω is a 3×3 lower-triangular matrix. The error term, $u_t^o \sim$ IID N(0,I). Once we construct our vector time series of macroeconomic variables, we use ordinary least squares to estimate ρ^o and Ω[1]. By doing so, we lessen the burden on our likelihood maximization (as will be discussed shortly).

Next, we estimate parameters related to our short rate equation. From (5), we can split δ_1 into its observable and unobservable parts, which allows us to rewrite r_t in the following manner.

$$r_t = \delta_0 + \delta_{11}' X_t^o + \delta_{12}' X_t^u \tag{13}$$

By treating the last term as an error, we can use ordinary least squares to derive δ_0 and a partial solution to δ_1 (the first 3 parameters). Performing these individual estimations has reduced the dimensionality of our likelihood maximization by $9 + 6 + 4 = 19$ parameters.

3.2 Restrictions

We assume that the unobservable variables take the following process:

$$X_t^u = \rho^u X_{t-1}^u + u_t^u \tag{14}$$

where ρ^u is a 3×3 matrix that is lower triangular and u_t^u is a 3×1 vector that is distributed normally with mean 0 and covariance matrix I. This specification for X_t^u comes originally from Dai and Singleton (2000) and is the most general specificiation for a Gaussian variable. Combined with our form for the observable macroeconomic component of X_t^o, this implies that μ is a zero vector.

Next we make the strong assumption that observable and latent factors are independent. By doing so, we can restrict the upper right and lower left 3×3 corners of Φ and Σ to be containing of only zeros. Furthermore, using (12) and (14) we designate the top left 3×3 corner of Φ to be equal to ρ^o and the corresponding bottom right corner, a lower triangular matrix containing six unknown paramters. Similarly the upper left 3×3 corner of Σ will be equal to Ω and the bottom right corner wil be a three-dimensional identity matrix.Similar to these these conditions, we will also assume that the upper right and lower left 3×3 corners of λ_1 will be restricted to zeros, maintaining our orthogonality constraint.

[1]To obtain Ω, we first calculate our errors in (12), $\epsilon_t^o = \Omega u_t^o = X_t^o - \rho^o X_{t-1}^o$. We then estimate the covariance matrix of our errors, $var(\epsilon_t^o)$, and use a Cholesky Decomposition: $var(\epsilon_t^o) = \Omega\Omega'$, for Ω a lower triangular matrix.

3.3 Likelihood Function

Despite our efforts so far, we still have a number of parameters to account for. To completely specify our model we need to find the following variables: $(\delta_0, \delta_1, \mu, \Phi, \Sigma, \lambda_0, \lambda_1)$. Following our preliminary estimations and restrictions, we have left to estimate (0, 3, 0, 6, 0, 6, 18), or a total of 33 free parameters. To estimate these unknowns, we will make use of maximum likelihood estimation.

3.3.1 Finding Unobservables using Measurement Errors

At this point of the estimation, we have a functional form for our yields (using (9), (10), and (11)), a time series of observable macro variables, and associated yield data (for yields corresponding to bonds of length $n = n_1, n_2, n_3...$). We know that for each n_i, we can write: $y_t^{n_i} = A_{n_i} + B_{n_i} X_t$. Suppose that we have m observable yields, k_o observable, and k_u unobservable factors. Also assume that $m = k_u$. By stacking all m yields, we can write the system compactly.

$$Y_t = A + BX_t = A + B^o X_t^o + B^u X_t^u$$

Here we allow $B = [B^o B^u]$, where B^o is the $m \times k_o$ matrix that only picks up the observable factors and B^u is the $k_u \times k_u$ matrix that only picks up the unobservables. Using the previous system, we can easily solve for X_t^u, as B^u is invertible.

As will be explained in the next section, the dimensionality of our observable yields is greater than that of our unobservable factors ($m > k_u$). To handle this situation, we look to the methodology established in Chen and Scott. We will assume that our vector of yields contains a measurement error, u_t^m, that is Gaussian with mean 0 and diagonal covariance matrix. In our data set of yields we have $m = 5$ and $k_u = 3$. This implies that we will have two yields with measurement errors. We rewrite our earlier system as:

$$Y_t = A + B^o X_t^o + B^u X_t^u + B^m u_t^m \tag{15}$$

In our particular estimation, B^o and B^u will be 5×3 matrices, and B^m is a 5×2 matrix. Because we assume that the measurement errors occur for only two yields, B^m will contain only two elements (one in each column in different rows) and zeros elsewhere. Given a choice of original parameters and B^m, we can form all coefficient matricies in (15). Next, we form the matrix $B^* = [B^u B^m]$, which is an invertible 5×5 matrix. To solve for X_t^u and u_t^m we rewrite (15) as the following system:

$$Y_t - A - B^o X_t^o = \begin{pmatrix} B^u & B^m \end{pmatrix} \begin{pmatrix} X_t^u \\ u_t^m \end{pmatrix} = \begin{pmatrix} B^* \end{pmatrix} \begin{pmatrix} X_t^u \\ u_t^m \end{pmatrix}$$

We can now invert to solve for our time series of 3×1 latent variables and 2×1 measurement errors.

3.3.2 Log-Likelihood

The likelihood function gives us a joint probability function for the occurence of bond yields and state variables. Our goal is to maximize this probability with respect to our unknown parameters. Equivalently, we will maximize the logarithm of the likelihood function, which provides the same solution and allows for ease in computation. The log-likelihood function, $\log L$, will take as input some vector of parameters, θ, and is as follows:

$$\log L(\theta) = \sum_{t=2}^{T} f(Y_t, X_t | Y_{t-1}, X_{t-1})$$

where $f(Y_t, X_t|Y_{t-1}, X_{t-1})$ is the conditional probability of yields and state variables at time t. Again using Chen and Scott, this is equivalent to:

$$\log L(\theta) = \sum_{t=2}^{T} -\log|J| + \log f_x(X_t|X_{t-1}) + \log f_u(u_t^m) \tag{16}$$

where $|J|$ denotes the determinant of the Jacobian matrix. f_x and f_u are the individual density functions for our state variable and measurement errors. We know our state variable is Gaussian and from assumption our independent measurement errors are also multivariate normal.

$$\begin{aligned}\log L(\theta) = &-(T-1)\log|J| - \frac{(T-1)}{2}\log|\Sigma\Sigma'| \\ &-\frac{1}{2}\sum_{t=2}^{T}(X_t - \mu - \Phi X_{t-1})'(\Sigma\Sigma')^{-1}(X_t - \mu - \Phi X_{t-1}) \\ &-\frac{T-1}{2}\log|Var_u| - \frac{1}{2}\sum_{2}^{T} u_t^{m\prime}(Var_u)^{-1}u_t^m\end{aligned}$$

where Var_u is the 2×2 diagonal covariance matrix of our measurement errors. Our Jacobian term, J, will be an 8×8 matrix given as:

$$J = \begin{pmatrix} I_{3\times3} & 0_{3\times3} & 0_{3\times2} \\ B^o & B^u & B^m \end{pmatrix}$$

3.3.3 Recap

As earlier mentioned, we need to estimate 33 free parameters to completely specify our model. In order to carry out this algorithm, we also need to designate 2 additional parameters for our measurement matrix, B^m, and 2 more for the standard deviations of our independent measurement errors (which will generate Var_u). This implies on total we have $\mathbf{33+2+2=37}$ free parameters. To summarize, given some vector, θ, of these parameters, we will:

1. Assign values from θ to all of our variables: $\delta_0, \delta_1, \mu, \Phi, \Sigma, \lambda_0, \lambda_1, B^m$, and Var_u.
2. Calculate the affine model coefficients (A_n and B_n for $n = 1, 2, 3, ..., 100$) using the recursive form in (9), (10), and (11).
3. Take coefficients corresponding to our yield data ($n = 1, 3, 12, 36, 60$). Stack and form A, B^o, and B^u.
4. Using bond yields data and constructed macro factors, invert (15) to calculate unobservable variables and measurement errors for each time period.
5. Combine the observable and unobservable factors to form our total state variable. Then calculuate the log likelihood function based on our choice of parameters and time series of state variables and measurement errors.
6. We repeat this process for different values of θ until we maximize our log likelihood value.

To this point we have duplicated the model derivation and estimation algorithm of the original paper. Now we will start to discuss the crux of our topic - the implementation of credit variables.

4 Data

4.1 Bond Yields

We have zero coupon yields of bond lengths $1, 3, 12, 36$, and 60 months[2]. For our estimation, we will be using data going back to Jan. 1990. While the original paper utilizes yield data that goes back to 1952, in implementing credit variables we would like to restrict ourselves to more recent data. Some of the key indicators we would like to utilize are only available starting from the mid-late 1980's. Further, it is needless to say that the lending environment over the last two decades is much different than it was over a half century ago.

4.2 Macro Data

4.2.1 Measuring the Credit Environment

Trying to pin down a single data series for the credit environment is a challenging and perhaps infeasible task. With large discrepancies between small private and retail credit markets, we need to choose data series that cover different sectors of the economy.

To measure ease in wholesale interbank-lending markets, we use the monthly spread between 3-Month US LIBOR, a measure of US interbank lending rates, and OIS, an overnight index swap rate which is correlated to the Federal Funds Rate (LIBOR-OIS). The LIBOR-OIS time series only goes back to the late 1980's and hence serves as a time restriction to our overall estimation. We next choose data series for consumer and business lending. For these components, we use yearly growth rates of total consumer credit (CONCRED) and total commercial and industrial credit (BUSCRED), respectively. We also use the M1 money multiplier (MULT), which serves

[2] $1-$month yields from Fama CRSP Data Files; all other yield data from Federal Reserve Board of Governors

Table 1: Principal Component Analysis, Credit

	1st	2nd	3rd	4th	5th
LIBOR-OIS	.254	-.551	.645	.445	-.136
CONCRED	-.466	-.394	-.508	.367	-.485
BUSCRED	-.401	-.621	.049	-.459	.491
MULT	-.538	.323	.194	.576	.487
CORPSPRD	.518	-.229	-.535	.354	.518
% Variance Explained	40.0	65.4	83.0	94.6	100.0

Table 2: First Component Correlations, Credit

LIBOR-OIS	CONCRED	BUSCRED	MULT	CORPSPRD
.360	-.659	-.567	-.761	.732

as a measure of velocity in the commercial banking system. Our last indicator will be the spread between the average AAA corporate lending rate and ten-year treasury rates (CORPSPRD)[3]. This indicator also measures unease in corporate lending markets. Following from the methodology of the original paper, we normalize each series to zero mean and unitary variance before extracting principal components.

Principle component analysis allows us to reduce the dimensionality and explain much of the variation in our original data set. By performing a linear transformation on our data, we receive again five "principal components" where the first principal component accounts for a large amount of the total variation - in our case 40% of the entire data set. Each principal component is a linear combination of all 5 data series; in Table 1 we report the coefficients of these combinations, or the "factor loadings."

To reduce the dimensionality of our state variable, we will simply take the first principal component. From table 1, it is evident that the first component loads positively on LIBOR-OIS and CORPSPRD, and negatively on CONCRED, BUSCRED, and MULT. Intuitively we can interpret positive movements of LIBOR-OIS and CORPSPRD as indicative of a weaker lending environment (as lending conditions are at greater unease). Similarly we can interpret negative movements of CONCRED, BUSCRED, and MULT as indicative of a weaker credit environment. Hence, this implies for positive shocks to the first principal component, it can be viewed as a negative movement in overall credit. To receive our credit factor, we take the negative of our the first component. From this point forward we will refer to this factor as "credit."

[3]LIBOR–OIS data from Global Financial Data; CONCRED and BUSCRED from Federal Reserve Board of Governors; CORPSPRD from Moody's Investor Services and Federal Reserve Board of Governors; MULT is calculated as the ratio between M1 money supply and the St. Louis Adjusted Monetary Base (see FRED)

Table 3: Principal Component Analysis, Inflation

	1st	2nd	3rd
CPI	.545	.826	-.146
PPIFG	.601	-.263	.755
PPICO	.585	-.499	-.639
% Variance Explained	87.5	98.2	100.0

Table 4: First Component Correlations, Inflation

CPI	PPIFG	PPICO
.863	.923	.942

4.2.2 Inflation and Real Economic Activity

We will similarly construct macroeconomic factors that relate to inflation and real economic activity. In the original paper the authors use the Consumer Price Index (CPI), Producer Price Index - Finished Goods (PPIFG), and Spot Market Commodity Prices (PCOM) as base inflation indicators. In place of PCOM we will be using Producer Price Index - All Commodities (PPICO)[4]. We follow the same procedure as we did for our credit variable and we receive the following results.

If we take a look at the loadings of the first principal component in Table 3, it is clear that the first component has a positive relationship with movements in consumer prices, producer prices of finished goods, and commodity prices; correlations are also very high for all three variables. For positive movements of this component, it is analogous to a positive shock to inflation. Hence we will interpret it as "inflation."

For real economic activity, I will use the Unemployment Rate (UE), growth rate of Employment (EMPLOY), and growth rate of Industrial Production (IP) [5]. The original authors use these indicators and also utilize the index of Help Wanted Advertising in Newspapers (I omit it here for simplicity). Again, we can easily interpret the first principal component of our analysis as a "Real Activity" factor. We see in Table 5 that it loads negatively on unemployment and positively on employment and industrial production. This implies for positive movements in this component, real economic activity increases.

4.2.3 Overview

We have constructed three macroeconomic factors (Real Activity, Inflation, and Credit) that will be used as the observable components of our state variable. Figure 3 displays the time series of these factors over the last two decades. We find that real activity and inflation are not tremendously

[4]CPI, PPIFG, and PPICO from Bureau of Labor Statistics; similar to Ang and Piazzesi we calculuate inflation rates as $\log{(I_t/I_{t-12})}$ where I is the corresponding inflation statistic.

[5]UE and EMPLOY from Bureau of Labor Statistics; IP from Federal Reserve Board of Governors; we calculate growth rates using 12-month log changes.

Table 5: Principal Component Analysis, Real Activity

	1st	2nd	3rd
UE	-.528	.750	-.398
EMPLOY	.642	.046	-.766
IP	.556	.659	-.506
% Variance Explained	75.5	96.3	100.0

Table 6: First Component Correlations, Real Activity

UE	EMPLOY	IP
-.795	.966	.837

correlated ($\approx .36$); similarly, credit and inflation are not highly linked ($\approx .27$). However, we find that credit and real activity show a moderately strong correlation ($\approx .70$). This will end up playing a role in our eventual results.

These factors allow us to explain some of the notable events of the last two decades, including the minor recessions of the early 1990's and 2000's and deflation scare of the early 2000's. Through the credit indicator we can also point out the buildup in credit preceding the technology and housing bubbles.

5 Results

Using the likelihood function defined by (16) we maximize the joint probability of bond yields and state variables with respect to a multi–dimensional (37×1) theta. Through many iterations of this algorithm we arrive at a θ that is suitable and fits the data moderately well.

5.1 Parameter Estimates

Implementing a numerical optimization of our likelihood function, we determine our estimates for all missing parameter values. As the likelihood surface was very rough, it took many iterations to find a local maximum that suitably corroborated with data. Due to the orthogonality constraints, there are blocks of zeros in many of our variables, notably Φ, Σ, and λ_1. Our parameters were as follows:

$$\mathbf{\Phi} = \begin{pmatrix} .997 & -.021 & -.002 & 0 & 0 & 0 \\ .058 & .921 & -.019 & 0 & 0 & 0 \\ .061 & -.024 & .941 & 0 & 0 & 0 \\ 0 & 0 & 0 & .160 & 0 & 0 \\ 0 & 0 & 0 & 1.065 & .662 & 0 \\ 0 & 0 & 0 & -.605 & -.511 & -.285 \end{pmatrix} \quad \mathbf{\Sigma} = \begin{pmatrix} .241 & 0 & 0 & 0 & 0 & 0 \\ .084 & .548 & 0 & 0 & 0 & 0 \\ .016 & .040 & .321 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\delta_{\mathbf{0}} = \begin{pmatrix} .295 \end{pmatrix} \quad \delta_{\mathbf{1}} = \begin{pmatrix} .004 \\ .004 \\ .087 \\ .911 \\ .314 \\ .156 \end{pmatrix} \quad \lambda_{\mathbf{0}} = \begin{pmatrix} .450 \\ .955 \\ 812 \\ -.878 \\ 2.218 \\ .069 \end{pmatrix} \quad \lambda_{\mathbf{1}} = \begin{pmatrix} -.108 & -.589 & .405 & 0 & 0 & 0 \\ .576 & .979 & -.583 & 0 & 0 & 0 \\ .180 & -.014 & -.172 & 0 & 0 & 0 \\ 0 & 0 & 0 & .181 & .275 & .091 \\ 0 & 0 & 0 & .669 & .205 & .533 \\ 0 & 0 & 0 & -.187 & 1.077 & -.020 \end{pmatrix}$$

$$\mathbf{B_m} = \begin{pmatrix} 0 & 0 \\ .150 & 0 \\ 0 & 0 \\ 0 & .219 \\ 0 & 0 \end{pmatrix} \quad \sigma(\mathbf{u_t^m}) = \begin{pmatrix} .0429 \\ .0336 \end{pmatrix}$$

5.2 Mean Yield Curve

Using these parameters we have a completely identified linear form for bond yields of maturity n at time period t (11). This means that given a state variable, X_t at each t, we can generate a yield curve for various $n = 1 : 60$. To test the effectiveness of our parameters, we take the average model-generated yield curve over the last twenty years (Jan. 1990 - Aug. 2010) and compare this curve to the function generated by average yields for $n = 1, 3, 12, 36, 60$ months.

Figure 4 displays the fit of average model results to data points. Our result is that the model effectively matches the data on average over the last twenty years. This basic result provides us validation to use these parameters as our model configures well to data.

5.3 Impulse Responses

We originally sought to examine the effect of credit shocks on bond yields. Using the parameter estimates and structural form for bond yields we can insert a shock into our state variable (specifically the macroeconomic components) and examine corresponding effects on the yield curve. We start with the basic form for our state variable (we leave out μ as it is a zero vector) and iterate backwards. Letting $N \to \infty$ and assuming that X_t starts at zero, we have that:

$$\begin{aligned} X_t &= \Phi X_{t-1} + \Sigma\varepsilon_t \\ &= \Phi(\Phi X_{t-2} + \Sigma\varepsilon_{t-1}) + \Sigma\varepsilon_t \\ &= \Phi^N X_{t-N} + \Sigma \sum_{i=0}^{N-1} \Phi^i \varepsilon_{t-i} = \Sigma \sum_{i=0}^{\infty} \Phi^i \varepsilon_{t-i} \end{aligned}$$

Using this form of X_t we can then express our continuous bond yields as:

$$y_t^n = \frac{A_n}{n} + (\frac{B_n'}{n})X_t = \frac{A_n}{n} + (\frac{B_n'}{n})\Sigma \sum_{i=0}^{\infty} \Phi^i \varepsilon_{t-i} \tag{17}$$

To calculate the impulse responses of bond yields, we apply a 1 standard deviation shock to our credit variable in ε_t at time period $t = 0$, assuming that our macroeconomic state begins at 0 and $\varepsilon_t = 0$ for all other t. We measure the effect of such a shock on short, medium, and long bond yields ($n = 1, 36$, and 60 months).

Figure 5 measures the effects of a $1 - \sigma$ shock on 1-month yields, for all macroeconomic variables. Figures 6 and 7 measure the effects for medium and long term yields. The most striking result from all three IR functions is that positive credit shocks raise bond yields. From the way we constructed our credit variable, we stipulated that any positive movement in this factor would constitute an easing in lending and positive flow of credit. Intuitively, we would expect that a more capacitative credit environment would lower rates on the short end. But in fact, our model results contradict our expectations. Earlier on we saw that our credit factor was highly correlated with our real activity factor. This ended up playing a key result as positive movements in real activity imply higher bond yields and so was the result for credit. We will later discuss further improvements on our measurement of credit flow.

A secondary result is that shocks to credit on all durations have a greater immediate impact on yields in comparison to shocks of real activity. If we examine Figure 8, we deduce that a shock to credit immediately shifts and quickly subsides the yield curve. Sixty months following the shock, the yield curve returns very close to steady state (when $X_t = 0$). If we similarly examine shocks of real activity, as in Figure 9, we find that the total shift of the yield curve is milder and more prolonged. Sixty months following the real activity shock, the yield curve has not yet returned to the steady state curve. In such a way we can interpret the effects of real activity to be milder and more persistent on bond yields than those of credit which are more immediate. This might indicate that shocks to real activity have more permanent, long term considerations while imbalances of credit have greater impact in the short term.

5.4 Other Considerations

Measuring credit flow was a difficult task as economic intuition did not match the results of our credit shocks. This begs us to ask, should we consider alternative constructions of our credit factor. By using a principal component analysis of different variables, it turned out that our eventual factor loaded strongly on CONCRED, BUSCRED, and MULT, variables that were significantly correlated to our real activity factor. In order to remove the real activity component from credit and get a “purer” estimate for this factor, we could look to extract a part of our current credit factor that is independent or orthogonal to real activity. Another way to do this would be to extract a principal component that is solely based on LIBOR-OIS and CORPSPRD. By doing so, we’ll get closer to examining the portion of credit that was at unease in the financial crisis and perhaps obtain a result that is richer for discussion.

While the model fits the data proficiently, the inflation shocks did not match economic intuition. Positive shocks to inflation in fact decreased bond yields - a chief drawback to our results. This underlies a much larger problem, regarding the maximium likelihood estimation of a high-dimensional problem on a rough likelihood surface. It often was the case that local maxima did not sufficiently fit data and that starting points for iterative algorithms were critical for achieving satisfactory results. While we succeeded in finding a decent local maximum, we cannot say with certainty that our parameter results were optimal. This in mind, I look forward to learning more about numerical optimization and implementing better techniques to receive a more complete solution.

6 Conclusion

In order to measure the effects of credit variables on bond yields, I estimate a six-factor Gaussian affine term structure model that incorporates both macroeconomic and latent variables. Following from Ang and Piazzesi, I construct my macroeconomic variables (real activity, inflation, and credit) using principal component analysis and estimate the parameters of my model using maximum likelihood estimation. By arriving at a parameter solution through a numerical optimization routine, I find that my model corroborates with zero coupon bond data for the last twenty years. By implementing impulse response functions, I arrive at my chief result that positive movements in credit flow actually increase interest rates. Further, shocks to credit are more temporary when compared to those of real activity which are milder and more permanent. While the results are interesting, some questions linger regarding the proper measurement of credit and validity of the maximum likelihood estimate. From these results we have gained some information that is potentially useful for policymaking. As credit shocks have a greater short term impact and are quicker to subside they might call for less attention from a policy perspective. Comparatively, real activity has greater consequences on the long term state of the yield curve and for this reason might demand closer attention.

From the debt-deflation hypothesis (Fisher(1933)) to the idea of the financial accelerator (Bernanke et al. (1996)) and beyond, much work has been done in understanding the effect of financial frictions and credit market movements on real economic activity. Given the impact of the financial crisis, there is only more room and need for research in this area. Through this exercise, we have been able to scratch at the surface of this issue. If nothing else, this project provided me a thrilling experience in learning about asset pricing, econometrics, and economic research in general. I look forward to improving on my results and pursuing other endeavors in research.

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[2] Backus, David, Silverio Foresi, and Chris Telmer, 1998, "Discrete-time models of bond pricing", NBER Working Paper No. 6736, September

[3] Chen, Ren-Raw and Louis Scott, 1993, "Maximum likelihood estimation for a multifactor equilibrium model of the term structure of interest rates," *Journal of Fixed Income 3*, 14–31

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[6] Sargent, Thomas and Lars Ljungqvist, 2004, *Recursive Macroeconomic Theory, 2nd Edition*, The MIT Press

[7] Vasicek, Oldrich, 1977, "An equilbirum characterization of the term structure," *Journal of Financial Economics 5*, 177-188

Figure 1: **Real Activity and Inflation (1990 - 2010)**

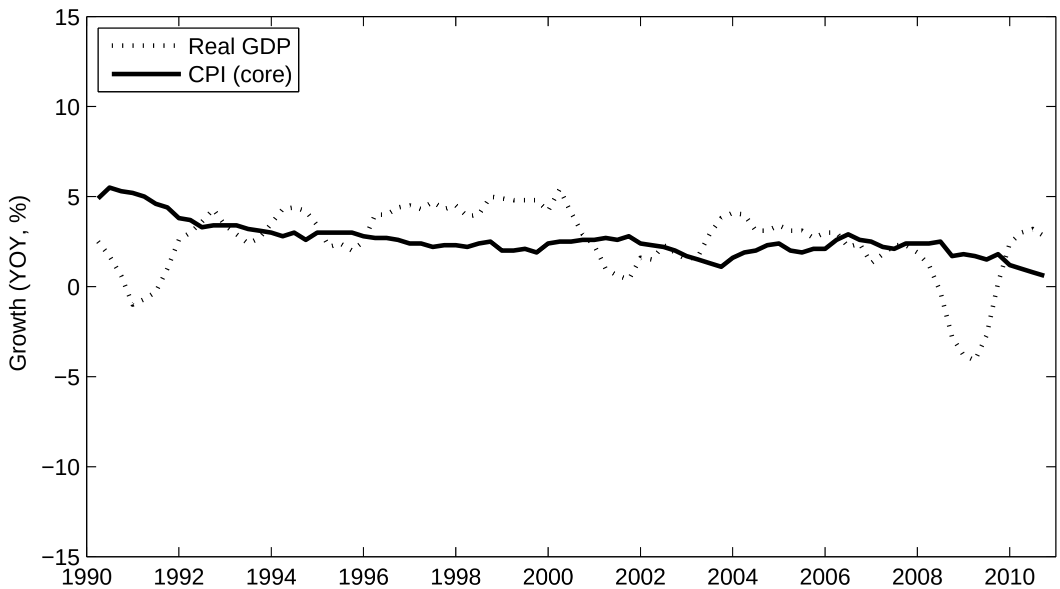


Figure 2: **Credit Conditions (1990 - 2010)**

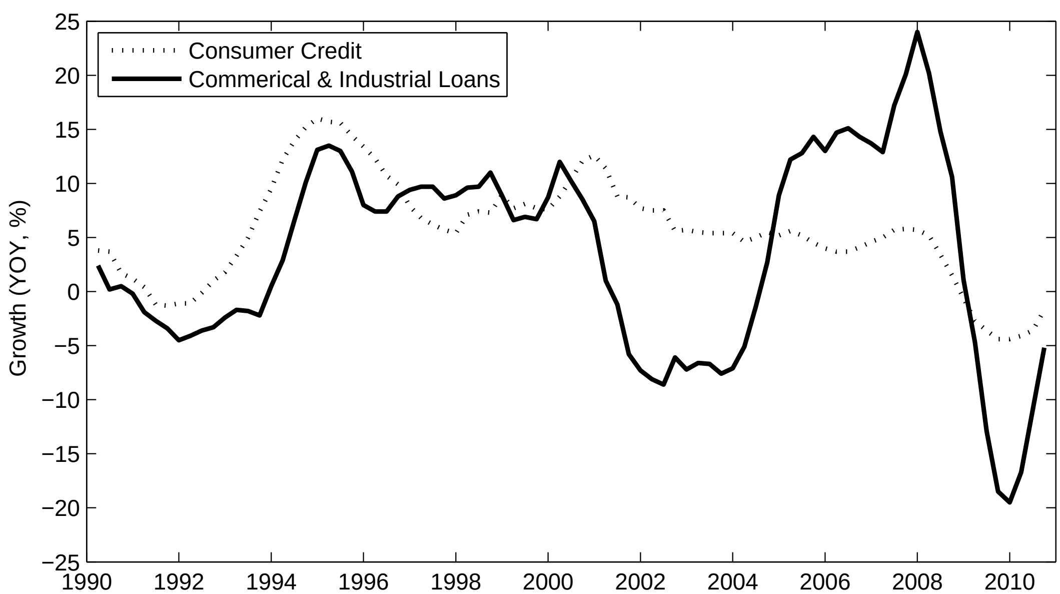
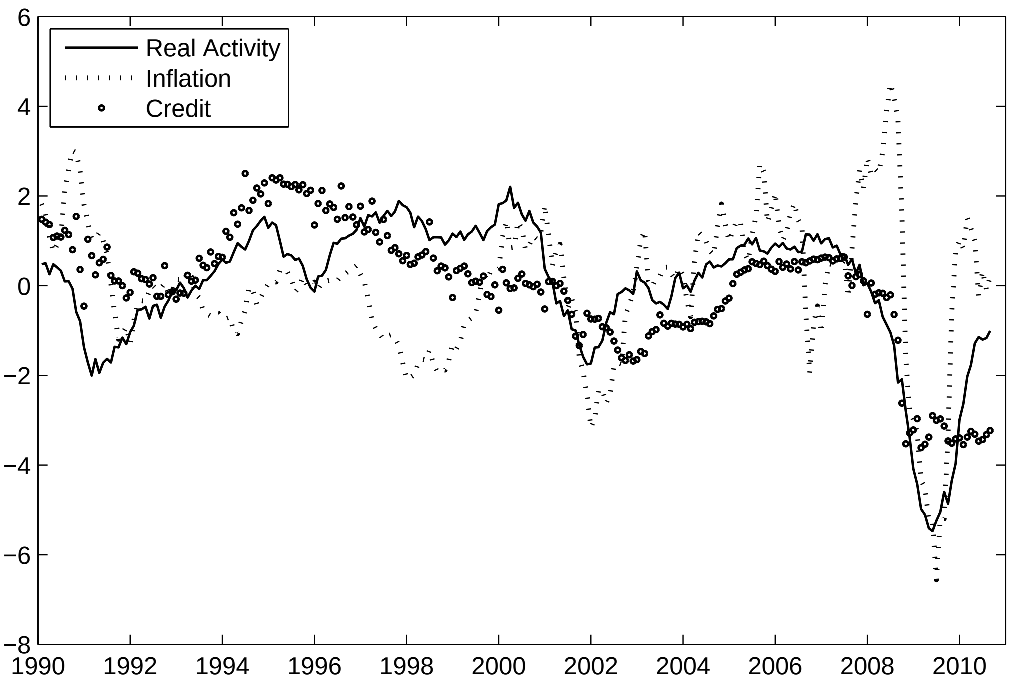
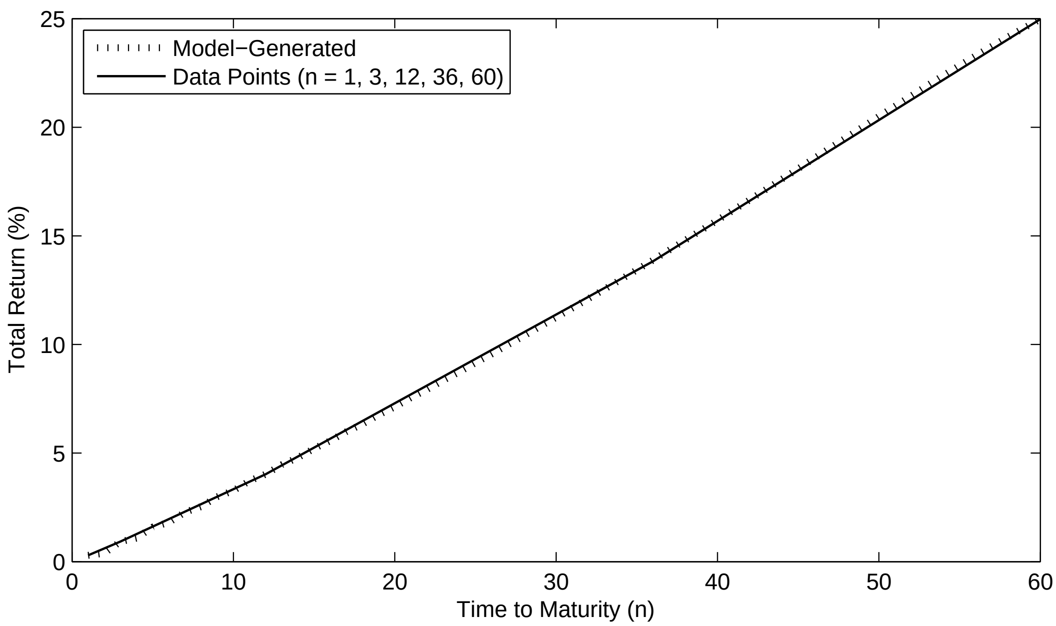


Figure 3: **Macroeconomic Factors (1990 - 2010)**



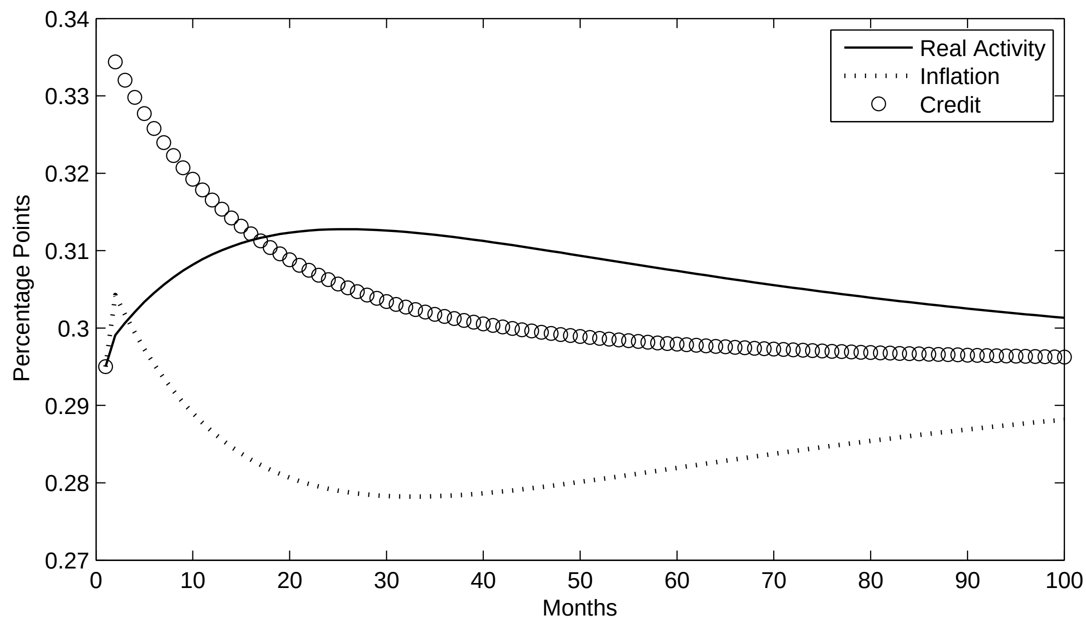
We construct three macroeconomic factors using a principal component analysis of various indicators related to each individual factor. Before running the PCA for each factor we normalize all data series to zero mean and unitary variance. All data is monthly from January of 1990 through August of 2010.

Figure 4: **Average Model Yields (1990 - 2010)**



Using the parameters defined in the results section we determine a yield curve for each time period t, $n = 1 : 60$. Averaging over all 248 time periods we receive the curve that is indicated by the tick-lines. We also conduct a similar process for our data points at 5 maturities, displayed by the solid line. We calculate total return as $(y^n \times n)$ where y^n is the average per-period yield over all time periods.

Figure 5: **Impulse Response, 1-Month Yield**



As discussed, we apply a $1 - \sigma$ shock to all of our macroeconomic variables. This graph measures the effect of the three separate shocks on the short rate. It is evident that an ease in credit lending actually raises interests rates.

Figure 6: **Impulse Response, 36-Month Yield**

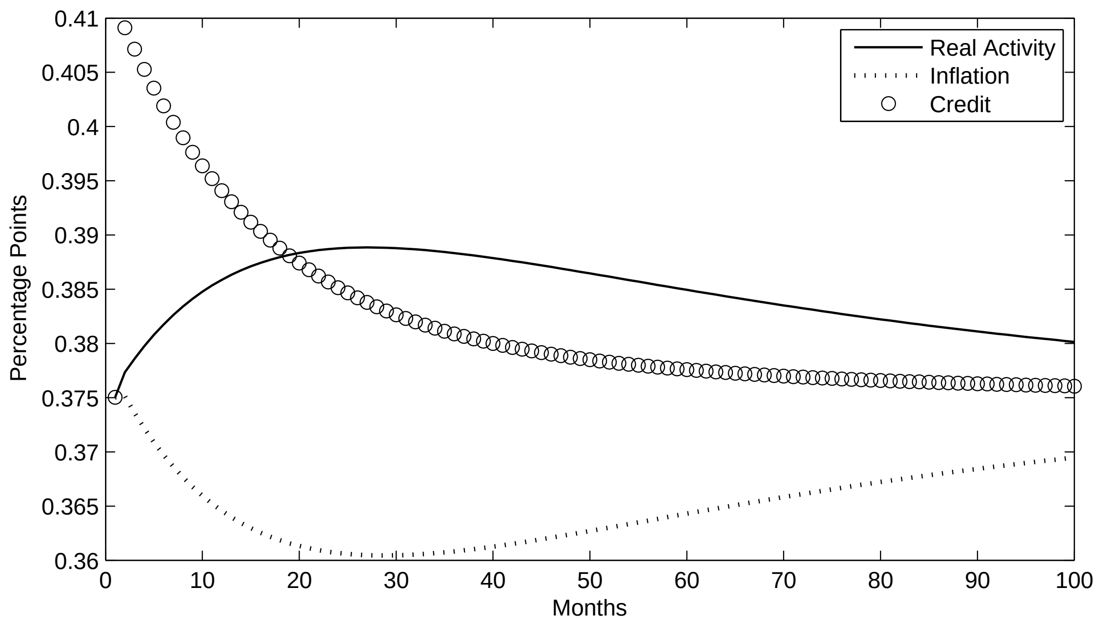
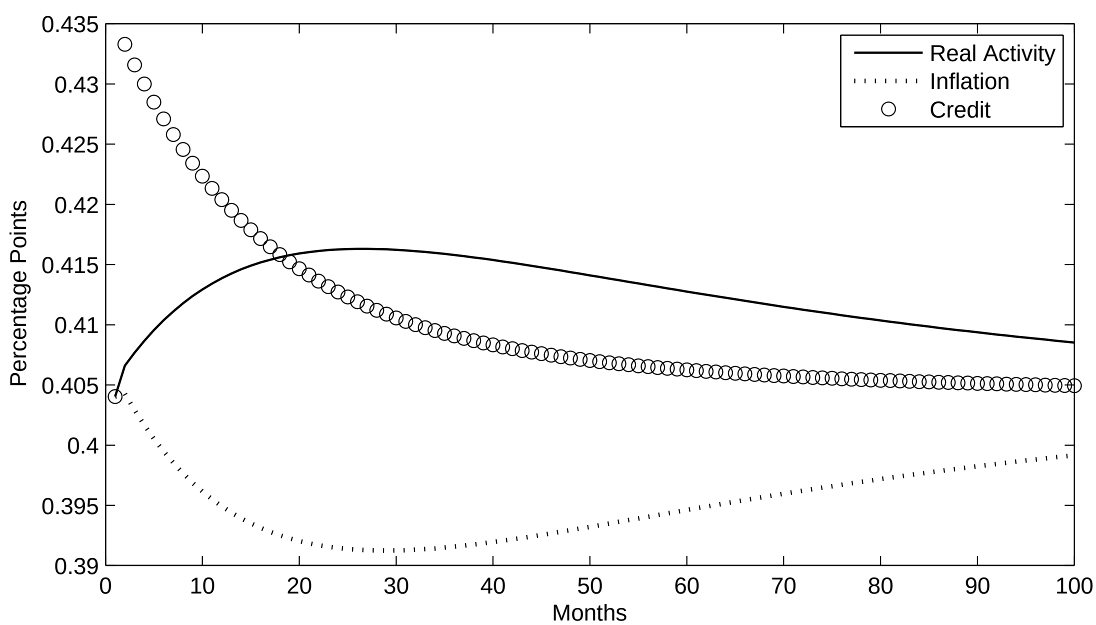
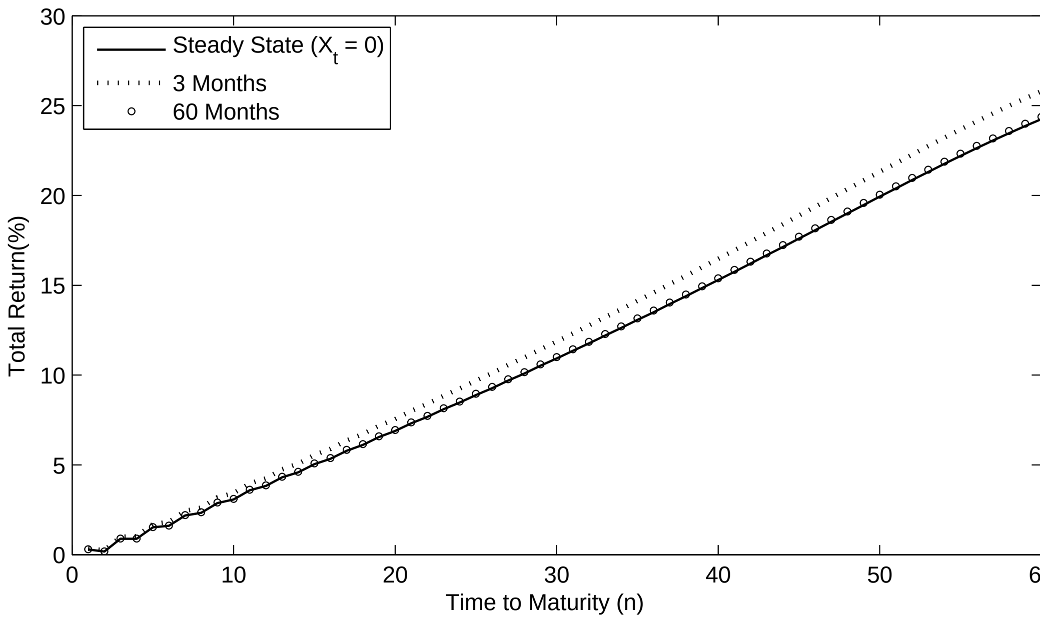


Figure 7: **Impulse Response, 60-Month Yield**



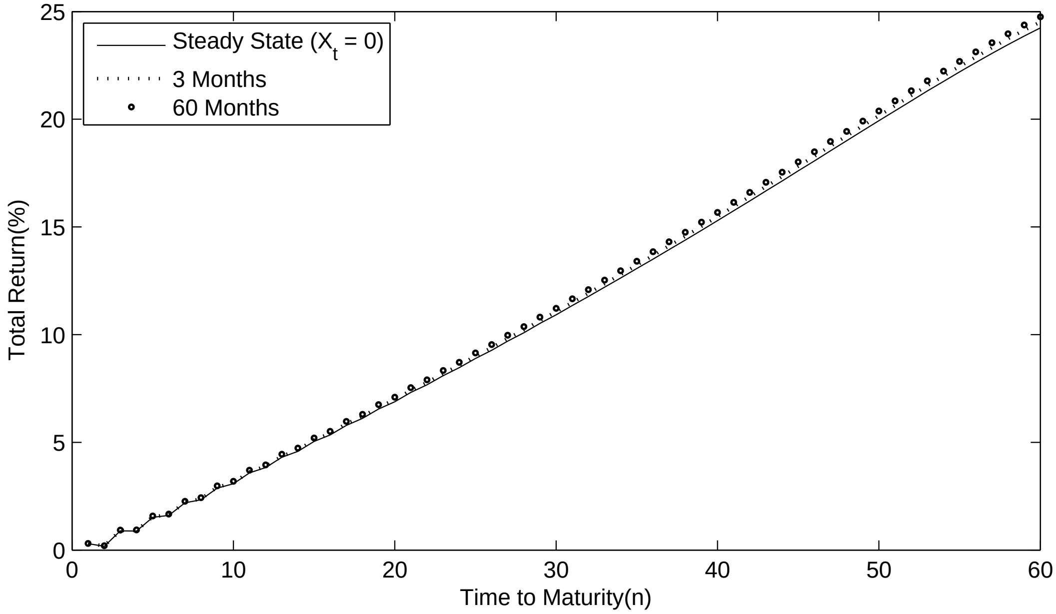
We similarly construct impulse responses for medium and long-term yields. Interestingly, the effects of macroeconomic shocks on these yields are very similar to those on the short interest rate. It is also worth noting that according to this model, an inflation shock lowers yields on the medium and long end - a result that doesn't match intuition.

Figure 8: **Yield Curve Following Credit Shock**



We display the effects of a 1-σ credit shock on the entire yield curve. We report both the yield curve at 3-months and 60-months following the credit impulse. Our steady state is the yield curve produced by $X_t = 0$, represented by the solid line. From this diagram it is apparent that the yield curve practically returns to steady state by 60 months following the credit shock.

Figure 9: **Yield Curve Following Real Activity Shock**



We display the effects of a 1-σ real activity shock on the entire yield curve. We report both the yield curve at 3-months and 60-months following the real activity impulse. If we closely observe, the yield curve does not return to steady state, even at 60 months following the real activity shock.