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Commodity Connectedness: Short-run Versus Long-run

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Abstract

In this paper I contribute to empirical literature that studies volatility spillovers among the commodity and equity market, focusing on short-term and long-term linkages. Studying the persistence of volatility transmission is helpful for understanding the information flow among markets, which is crucial for risk management and regulators. I evaluate rich dynamics of connections between equity and commodity sectors, showing that the global financial crisis 2008-2009 led to unprecedential spillovers from equity to commodity markets with high persistence while the collapse of Oil prices in 2014 led to strong and long persistent spillovers from the energy market to the equity market.

1 Introduction

Over the last decades, the connection between different assets has risen on their importance as the world tends to be more and more interconnected. For this reason, analysing spillovers between markets have become important for many areas of research such as risk management, portfolio theory or regulations design and policy analysis. The global crisis of 2007-2008 showed that failure of one market can quickly propagates to other markets as well. In this thesis, I investigate volatility spillovers among the commodity market and equity, as the recent financialization of the commodity market has risen a question whether the involvement of institutional investors and traders in commodity market have changed the nature of market. According to the U.S. Commodity Futures Commission (CFTC 2008) institutional investors increased the capital invested in commodity-related instruments from approximately 15 billion in 2003 to more than 200 billion in mid-2008 (Tang and Xiong (2012)). Up to now, several studies investigate whether since 2007-2008 the connections between commodities and the equity market have increased. This is partially investigated by Creti et al. (2013) who found out that several soft commodities (namely oil, coffee and cocoa) became more connected with US stock market in the period after the financial crisis than ever before.

Studying volatility connections of asset returns has a long tradition. Various empirical methods have been developed to capture the complex behaviour of volatility of financial time series. Since Sims (1980) variance decomposition has become a profound way of identification of volatility sources. Dibold and Yilmaz (2009) created general methodology that uses variance decomposition of vector autoregressive process to measure the share of variance that is explained by other variables. The approach has become quickly very popular for its generality and simplicity. Diebold and Yilmaz also created several studies that are devoted to different markets - Stock markets, foreign exchange markets, commodity markets and others and they launched a web page where connectedness is measured in real time [1].

Although measures of Diebold and Yilmaz gives valuable information about aggregated connectedness of system and about the direction of information flow, they do not distinguish between shocks in volatility that quickly disappears and shocks, that are long lasting. Why should evaluate the persistence of volatility transmission? The answer lies in investors preferences as they may have various investment horizons, which creates linages with various duration. Thus not only the magnitude of connections but also their persistence should be taken into account.

Baruník and Křehlík (2018) deal with this problem by developing new generalized framework built on Diebold and Yilmaz (2012) and Stiassny (1996). They show that a variance decomposition that is produced in time domain, can be extended into frequency domain using spectral representation of time series, which allows to distinguish between connectedness with short, medium and long duration. They illustrate how the method works on returns of S&P 500 in the period from 2001 to 2015 showing that the nature of connectedness changed dramatically with financial crisis, turning from short-run connections in volatility to long-run.

In this thesis, I contribute to the empirical literature that analyses the volatility transmission among commodities and their links to the equity market. The aim of this paper is to identify how persistent the transmission of uncertainty on commodity market is. More specifically, I use methods designed by Baruník and Křehlík (2018) to analyse volatility connectedness in a set of commodities including the Standards and Poor's 500 index to cover the impact of financial markets on different commodities. As argued in Baruník and Křehlík (2018) we can consider volatility spillovers as proxy for uncertainty transmission through the market. For the analysis, I use the most widely traded commodities Copper, Gold, Silver, Corn, Cotton, Crude Oil and Natural Gas to represent different types of commodities: precious metals, industrial metals, agricultural commodities and energies. Because S&P 500 include 500 largest U.S. companies from various industries

[1] financialconnectedness.org

and selected commodities represents all major commodity sectors, they should be a good approximation of connections among the commodity market and the equity. Our data sample covers the period from 4 January 1993 to 31 December 2015 thus I have enough observations to evaluate changes in volatility connectedness over the time including the effect of the global financial crisis.

I believe that in our globalized world it is crucial to understand the nature of spillovers among stocks and commodities because volatility spillovers are crucial for the assessment of undertaken risk. Moreover, increased interconnectedness of assets might be a first sign of a potential financial crises an noted by Diebold and Yilmaz (2012). As I have argued before, not only the magnitude of spillovers but also their persistence should be assessed by investors and regulators. In our analysis I pay particular attention to the recent 2007-2008 financial crisis and evaluate the impact on of financial turmoil on volatility among the commodity market and the stock market and ask whether the nature of connections have changed. Specifically, I use rolling sample over 250 days and look at the evolution of estimated short-run and long-run connectedness. I explore connections between different commodities and the equity index and answer the question if the level of persistence of connections differs and how they develop. To our best knowledge this topic has not been sufficiently addressed by existing literature, yet. The results of this paper might be interesting for variety of agents operating of financial and commodity markets, such as risk managers, investors but also regulators and institutions and should help to understand the frequency dynamics of connections that remain hidden when conventional measures of connectedness are used.

The remaining part of the paper is structured as following. Chapter 2 gives an overview of existing literature that relates either to the transmission of volatility between commodities end equity or to spillovers within the commodity market. Chapter 3 is devoted to derivation of methodology. In this Chapter I explain concepts of Diebold and Yilmaz (2009, 2012); Diebold and Yılmaz (2014) further extended by Baruník and Křehlík (2018) into frequency domain. Building on the vector autoregressive setting I show how to use variance decomposition for evaluation of linkages between assets. Furthermore, I use the Fourier transform to derive variance decomposition for given frequency of shocks. This result is used to construct the measure of connectedness, which allows us to measure spillovers on any frequency band. A detailed description of our data set can be found in Chapter 4. In Chapter 5 I investigate the transmission of uncertainty between different types of commodities as well as volatility spillovers from the stock market to selected commodities and the other way around. Finally, in Chapter 6 I summarize our findings, discuss the results and suggest possible extensions of our analysis.

2 Literature review

Over the last few decades, as the commodity market and financial market have become more interconnected, numerous researcher have studied linkages among commodities, financial markets, currencies and global economy.

A large volume of empirical studies considers the relationship of Oil and global economy, as Oil is the key source of energy and important an production factor for various industries. Hamilton (1983) investigates connections between US economy and Oil prices, showing statistically significant relationship between the increase in price of Oil and declining output of US economy several quarters after as the US is net importer of Oil. Since then researchers have studied the impact of price of oil on US economy. In Hamilton (1996) he further expand his studies on the period that covers the invasion to Kuwait and tested for Granger causality, founding evidence of asymmetric reaction of economy on shocks into oil. Recently, Hamilton (2009) investigates causes of shocks in prices of oil in 2007-2008 and its implications for US economy concluding that the shock in Oil prices was caused mainly by increasing consumption in energy intensive industries that raised prices.Rotemberg and Woodford (1996) quantify the effect of Oil on U.S. GDP proposing that 10 percent increase in oil prices should cause decline in GDP about 2,5 percent on average after approximately year and half.

More recently, various studies focus on volatility spillovers from the oil to key stock markets. Arouri et al. (2011) uses VAR-GARCH model to study volatility transmission between the oil market and European and US financial markets with respect to industry sectors. They use VAR(1)-GARCH(1,1) model proposed by Ling and McAleer (2003) to capture the conditional volatility and cross effects. Their results indicate significant information transmission from Crude oil to European stock markets, whereas in case of US stock markets the effects are mutual. Apart from that, the aim of their paper is to derive optimal weights for diversification purposes in oil-stocks portfolio. They also show that optimally hedged portfolio, by taking short position in oil futures, has better performance than traditional portfolio. In Arouri et al. (2012) they focus on European equity market and oil market connections. Applying the same VAR-GARCH framework they conclude that industries differ in a magnitude of connections with oil prices, therefore in calculation hedging ratios they get considerable different results for each industry. They also show that, in general, volatility on European stock market does not effect volatility of oil prices, on the other hand volatility of oil transmit to stock market volatility which is in line with their previous findings. Degiannakis et al. (2013) who buids on the work Kilian (2009) Kilian and Park (2009) analyses more deeply the sources of shocks into oil prices and their spillovers to number of industrial sectors using time-varying heteroscedastic environment. Their results indicate that both source of shock and type of industry matter. More specifically, an increase in prices which is caused by a higher aggregate demand causes higher volatility of European stock market, whereas shocks that arise from supply-side do not effect volatility of the market. Maghyereh et al. (2015) investigates the connections between the oil implied volatility and the implied volatility of equities from eleven major stock markets in the period that followed the 2008 financial crisis in Europe They showed transmission of volatility from oil to equity market dominates the other way around. Moreover, in Maghyereh et al. (2016) authors claim there is an important transmission of volatility from oil market to stock markets in Middle East.

Following literature considers spillovers among the commodity market. Large part of literature studies how shocks into energies propagates through the commodity market. Nazlioglu et al. (2013) investigates connection between oil and agricultural commodities, concretely wheat, corn, soybeans, and sugar over the time period that covers the financial crisis and analysed changes in generalized impulse responses functions before and after the crisis. They conclude that the risk transfer was negligible before the crisis. Since then, shocks from Oil have transmitted to agricultural commodities. Their results as well as findings of Du et al. (2011), who investigate volatility linkages, implies that linkages between energies and agricultural commodities have become stronger in recent years.Ji and Fan (2012) use E-GARCH model to evaluate volatility spillover from crude oil to other commodities showing that crude oil has a core position in commodity market. They give an evidence that crude oil is a source of volatility that spillovers into other commodities.

Diebold et al. (2017) analyse volatility linkages in a set of 19 commodities including energies, industrial and precious metals, fibres and agricultural commodities, livestock and exotic over the period from 2006 to 2016. They work with range-based estimates of volatilities to shock volatility connectedness within the commodity market, arguing that they are almost as efficient as measuring realized volatilities. They show that commodities are grouped according to sectors: agricultural, livestocks, energies, precious metals and non-ferrous metals. Cotton, cocoa and coffee, usually called softs, are considered as a residual category, thus they are more independent.

Several studies devote to the effect of portfolio investment into commodities and equity. Tang and Xiong (2012) investigate implications of increasing demand for investing in commodity indexes, concluding that fast growth of investment into commodities result in higher correlations between commodities and equity indexes. They argue that prices of commodities are no longer determined only by balance of its supply and demand but more and more depends on behaviour of investors who operate on commodity market. Other studies provide evidence of financialization of commodities as well. Mensi et al. (2013) uses VAR(1) - GARCH(1,1) model to investigate connections in returns of selected commodities representing commodity market and S&P 500 and volatility transmission among them. In particular, they analyse connections among WTI, Brent, Gold, Wheat, Beverage and index S&P 500, concluding that past shock and uncertainty on the stock market have rich impact on gold and oil. Hamadi et al. (2017) investigate volatility spillovers among agricultural commodities, namely corn, wheat, soybean and soybean oil over the period from 1999 to 2015. They deploy the combination of following econometric models: ICSS, GARCH(1,1) and 3SLS and studied agricultural commodities with respect to financialization of commodities. Their results indicate that macroeconomic announcements have significant impact on volatility of agricultural commodities with the highest impact on corn. Creti et al. (2013) study relationship between 25 commodities, using the Commodity Research Bureau Index and S&P 500 Index over the period from the year 2001 to the year 2011. Using dynamic conditional correlation GARCH model introduced by Engle (2002) they found that correlation between stocks and commodities differed over the time period, especially during financial crisis when they observe high correlations. Some commodities seem so be more connected with stock market than others, especially oil, coffee and cocoa. Moreover, their results indicate correlation with S&P 500 grows when stock market is on rise. Kang et al. (2017) deploy DECO-GARCH model to examine spillovers among six commodities that covers connections between energies, agricultural commodities and precious metals. Using the framework developed by Diebold and Yilmaz (2012) they investigate information transmission among selected commodities with respect to the direction of information flow. They used weakly closing prices of Gold, WTI, Wheat, Rice, Corn and Silver form the year 2002 to 2016 and using rolling sample over 104 week they calculated both return spillovers and volatility spillovers. Their results indicate that connectedness among selected commodities reach the highest level at the end of the second European debt crisis that started in May 2011. The final part of their study they devoted to implication for hedging strategy and portfolio optimization.

Literature that considers the frequency dynamics of volatility spillovers is quite limited. Lau et al. (2017) devote their paper to return spillovers among white precious metals ETFs, gold, oil and equity market using daily returns of gold, silver, platinum, palladium, oil and global equity during the period from 2006 to 2016. They used framework developed by Baruník and Křehlík (2018) to analyse the frequency dynamics of spillovers considering 4 categories with respect to their persistence. They result shows that connections have mainly short-run character, with persistence shorter than one week. Furthermore, they use E-GARCH model and to capture non-linearities in the volatility of selected white precious metals. They conclude that silver and platinum shifts from industrial metals more to investment assets and confirm the hypotheses that gold is the main source of spillovers to white precious metals. Krehlik and Barunik (2017) devote to shocks into volatility of oil supply and demand with respect to the persistence of shocks. They assume that shocks to system that arise from crude oil represents supply side shocks whereas Gasoline and Heating oil represent the demand side. Their findings indicate that in the period from 1990 to 2016 supply side shocks to volatility prevails, during years 2006-2011 demand side shocks dominate the other way around, having mostly long-term effect on volatility of crude oil.

3 Methodology

In this part, I describe the theoretical background that I use for estimation of frequency dependent-connectedness of assets. Firstly, we establish the vector autoregressive model (VAR) and Wold decomposition that is used for construction of impulse response functions. I than follow the approach of Diebold and Yilmaz and define general forecast error variance decomposition as it is described in Diebold and Yilmaz (2012). Based on the general forecast error variance decomposition I show how are defined connectedness measures. Finally, I introduce the Fourier transform of an impulse response functions and use it to analyse frequency dynamics of GFEVD as proposed in Baruník and Křehlík (2018). The transformation of GFEVD will tell us what share of variance in each of variables accounts to which variable with respect to frequency. Following the work Baruník and Křehlík (2018) I show how to decompose the aggregated connectedness to short-run and long-run components.

Establishing variance decomposition of vector autoregresive process

As a building block for our theory I let us have a vector of time series variables $X_t = (x_{1,t}, ..., x_{N,t})$ described vector autoregressive stationary process of k-th order:

$$X_t = \Phi_1 X_{t-1} + \Phi_2 X_{t-2} + ... + \Phi_k X_{t-k} + \epsilon_t, \tag{1}$$

where Φ_i, $i = 1, ..., k$ are $N \times N$ matrices of unknown coefficients and ϵ_t is white noise with covariance matrix Σ. Furthermore, I assume that $E(x_t) = 0$, thus the data are centralized. I call the random variable ϵ_t innovation because it is assumed to be independent on previous values in other words unpredictable. Our VAR model has the advantage that can be estimated by the classical OLS procedure. To get the notation simpler I define lag operator that moves indexes back for one unit (day) as

$$LX_t = X_{t-1} \; . \tag{2}$$

$L^j X_t$ is equal to X_{t-j} and I can define a lag polynomial which is an operator that takes present value and return any combination of past values of the time series as

$$\Phi(L) = \Phi_0 + \Phi_1 L + \Phi_2 L^2 ... \Phi_k L^k \; . \tag{3}$$

Using this notation I can now rewrite our model as follows

$$X_t = \phi X_{t-1} + ... + \phi X_{t-k} + \epsilon_t = (\Phi_0 + \Phi_1 L + ... + \Phi_k L^k) X_t + \epsilon_t = \Phi(L) X_t + \epsilon_t \; , \tag{4}$$

which gives us simpler notation that I use in the rest of the thesis. Just note that the lag polynomials can be either finite or infinite. Using the lag polynomials the vector autoregressive moving average process $ARMA(k, l)$ and $MA(l)$ process can be defined as

$$\Phi(L) X_t = \Psi(L) \epsilon_t \; , \tag{5}$$

$$X_t = \Psi(L) \epsilon_t \; . \tag{6}$$

An interesting way how to characterize our time series is to construct a moving average representation which is only a linear combination of independent innovations. According to the Wold theorem, a stationary VAR process of order k can be represented by $MA(\infty)$ as

$$X_t = \sum_{i=0}^{\infty} \Psi_i \epsilon_{t-i} = \Psi(L) \epsilon_t \; , \tag{7}$$

where $MA(\infty)$ coefficients $\Psi(L) = (1 - \Phi(L))^{-1}$, so $\Psi(L)$ obey a recursive formula: $\Psi_i = \Phi_1 \Psi_{i-1} + ... + \Phi_p \Psi_{i-p}$, with Ψ_0 identity matrix and ϕ_i as in (1).

Wold representation is widely used in time series application as it gives us decomposition of present value of time series on past innovations. One advantage of MA representation is that I can now simply calculate an impulse-response functions, because MA representation and Impulse response function are equivalent. [2]

I would like to model connections between assets by impulse-response. Let us first consider that covariance matrix Σ is diagonal, which means that shocks to variables are uncorrelated at any time. Impulse response function answer the question, how unit shock to i-th variable propagates through the system in time, assuming other things are equal. Important assumption is that there is covariance matrix of white noise is diagonal, so the source of shock can be identified. However, in most of cases the covariance matrix Σ is not diagonal, that means innovations effecting variables are supposed to be correlated. The origin of shock is therefore not clear and it does not make any sense to ask what is the *cetiris paribus* effect of shock in one of the assets on others, because shocks usually comes together and effect more than one variable.

The initial connectedness measure developed by Diebold and Yilmaz (2009) is based on Choleski decomposition proposed by Sims (1980) which orthogonalize shocks in variables. However, proposed method in not order invariant, thus can give us different results for different orders. Kloessner and Wagner (2013) shows that using different orders may give result that are underestimated or overestimated about several percent.

These limitations were addressed in Diebold and Yilmaz (2012). They developed a new framework that builds on Pesaran and Shin (1998) and Koop et al. (1996) whose methods are used to construct order-invariant forecast error variance decomposition, based on generalized impulse-response and they defined directional spillovers from one asset to market as well as the other way around.

The *generalized impulse response function* (GI) is defined as the difference between conditional expectation of future value conditional on current shock and past and conditional expectation of future value given only history. Following definition establishes the generalized impulse response function in $MA(\infty)$ representation as proposed by Pesaran and Shin (1998)

$$GIRF_j(h) = \Sigma_{jj}^{-1/2} \Psi_h \Sigma e_j \ , \tag{8}$$

where e_j is a selection vector, Σ is variance matrix and Ψ_h is coefficient from MA representation.

To continue, with respect to Diebold and Yilmaz (2012) and Pesaran and Shin (1998) I define *general forecast error variance decomposition* (GFEVD), which represents share of H-steps ahead forecast error variance decomposition of i-th variable that is caused by a shock in variable j. For $i = j$ I call it own variance share and cross variance share otherwise.

Definition 1. *Let us have a stationary random process. General forecast error variance decomposition (GFEVD) is defined as*

$$(\theta_H)_{i,j} = \frac{\Sigma_{j,j}^{-1} \sum_{h=0}^{H} (\Psi_h \Sigma)_{i,j}^2}{\sum_{h=0}^{H} (\Psi_h \Sigma \Psi_h^T)_{i,i}} \ , \tag{9}$$

where Ψ_h represents the h-th coefficient of $MA(\infty)$ representation, $\Sigma_{j,j}$ represents j-th diagonal element of covariance matrix Σ

The notation of $(\theta_H)_{i,j}$ is taken from Baruník and Křehlík (2018) where detailed derivation of GFEVD can be also found. In practical applications the prediction horizon is usually in range 5 - 20 days. However, taking horizon H for instance 10 days does not mean I look at connections with persistence of 10 days. Note that GFEVD is not a

[2] detailed description of impulse-response can be found in Cochrane (2005)

normalized quantity, thus to get a percentage share on variance, one needs to standardize the quantity defined above.

Definition 2. *Standardized GFEVD is defined as*

$$(\tilde{\theta}_H)_{i,j} = \frac{(\theta_h)_{i,j}}{\sum_{i=1}^{n}(\theta_h)_{i,j}} . \tag{10}$$

From the above definition $\sum_{i=1}^{n}(\tilde{\theta}_h)_{i,j} = 1$, so I can think $(\tilde{\theta}_h)_{i,j}$ represents the percentage share of H-steps ahead error variance in i-th variable that accounts to shock in j-th variable.

To sum up, for now have defined VAR setting and constructed general forecast error variance decomposition based on moving average representation of vector autoregressive process - representation that decompose historical values of X_t on an infinite linear combination of independent innovations, which is the same as impulse-response function of the process. Then I have followed the approach of Diebold and Yilmaz (2012) and constructed variance decomposition that is independent on ordering of variables. In the following section I construct the standard Diebold and Yilmaz measures of connectedness.

Establishing connectedness measures in time domain

Here I continue with definition of connectedness table and the total connectedness, total directional connectedness from i-th variable to other variables and total directional connectedness from other variables to i-th variable.

Based on the general forecast variance decomposition I can simply define pairwise directional connectedness as the share of variance i-th asset that accounts to j-th asset. Using the notation from Diebold and Yılmaz (2014) let us define pairwise directional connectedness from j to i as

$$C^H_{i \leftarrow j} = (\tilde{\theta}_H)_{i,j} . \tag{11}$$

By connectedness matrix I understand matrix of pairwise directional connectedness.

Net pairwise connectedness is obtained as net contribution to volatility of j-th asset from i-th asset

$$\Delta C^H_{i,j} = C^H_{j \leftarrow i} - C^H_{i \leftarrow j} . \tag{12}$$

According to Diebold and Yılmaz (2014) total connectedness of the system can be defined as the sum on non-diagonal elements of connectedness matrix over N, because they represent what share of variance is explained by shocks into other variables. Following the notation from Diebold and Yılmaz (2014) let us define *the total connectedness* or *overall connectedness* as

$$C^H = \frac{\Sigma^N_{i,j=1, i \neq j} \theta^H_{i,j}}{\Sigma^N_{i,j=1} \theta^H_{i,j}} . \tag{13}$$

I can also look at the share of innovation that i-th variable receives from other variables. I call it directional connectedness from others to i (or shortly FROM connectedness)

$$C^H_{i \leftarrow \bullet} . \tag{14}$$

On the other hand I say that directional connectedness from i to others is

$$C^H_{\bullet \leftarrow i}, \tag{15}$$

shortly TO connectedness.

Finally, I define the NET directional connectedness as the difference between $C^H_{\bullet \leftarrow i}$and $C^H_{i \leftarrow \bullet}$, which is probably the most interesting quantity. It says what is the net contribution of variable to volatility of the system and therefore the source of uncertainly that spills to others.

Table 1: Connectedness Table

	x_1	x_2	...	x_N	FROM Others
x_1	-	$C^H_{1\leftarrow 2}$	...	$C^H_{1\leftarrow N}$	$C^H_{1\leftarrow \bullet}$
x_2	$C^H_{2\leftarrow 1}$	-	...	$C^H_{2\leftarrow N}$	$C^H_{2\leftarrow \bullet}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
x_N	$C^H_{N\leftarrow 1}$	$C^H_{N\leftarrow 2}$	...	-	$C^H_{N\leftarrow \bullet}$
TO Others	$C^H_{\bullet\leftarrow 1}$	$C^H_{\bullet\leftarrow 2}$	...	$C^H_{\bullet\leftarrow N}$	C^H

The connectedness table or the spillover table, summarize above defined measures of connectedness (Table 1). TO spillovers and FROM spillovers are obtained as a sum of columns and rows respectively.

To sum up, GFEVD gives us N $\times$ N matrix of pairwise directional connectedness, that represents a share of variance in i-th variable that is associated with j-th variable. If I calculate the share of non-diagonal elements I get the total connectedness of the system. Then I have $2N$ measures of total directional spillovers or total directional connectedness. Finally, I can calculate the net pairwise directional connectedness and net total directional connectedness, which indicates which asset is a net giver of volatility to the system or which variable a is net receiver of volatility from the system.

Connectedness measures in frequency domain

In the previous section I defined connectedness measures in the time domain which do not distinguish between short-run and long-run connections. In this section, I shed light on the construction of frequency dependent measure of connectedness. Firstly, I introduce Fourier transform that I use to define variance decomposition on a frequency band. Secondly, building on the general forecast error variance decomposition and connectedness measures defined in the previous section I decompose the connectedness measure on the short-run and long-run connectedness.

Fourier transforms and spectral representation

Definition 3. *Let us take and any real discrete series $\{x_t\}^{\infty}_{-\infty}$. The Fourier transform of the series $x(\omega)$, $\omega \in (-\pi, \pi)$ is defined by the following equation*

$$x(\omega) = \sum_{t=\infty}^{\infty} e^{-i\omega t} x_t \; , \tag{16}$$

where i denotes the complex unit.

using identity $e^{it} = cos(t) + isin(t)$ I can rewrite the Fourier transform as a sum of "weighted" sine and cosine in complex domain,

$$x(\omega) = \sum_{t=-\infty}^{\infty} cos(\omega t) x_t - isin(\omega t) x_t \; . \tag{17}$$

I can recover back the time series x_t using the inverse Fourier transform. The following formula defines the inverse Fourier transform as

$$x_t = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{+i\omega t} x(\omega) \; . \tag{18}$$

It can be easily showed that if I plug in the definition of $x(\omega)$ and exchange the order of sum and integral, I get the original series x_t. Detailed derivation can be found in Cochrane

(2005). The intuition behind is that Fourier transform is an infinite combination of sine and cosine waves, hence the real part of $x(\omega)$ is viewed as a infinite combination of periodical functions. Note that generally, the Fourier transform of series is in complex domain, but for symmetric series such that $x_t = x_{-t}$ the Fourier transform gives us real quantity.

Following definition establish spectral density, which is the Fourier transform of autocorrelation function

Definition 4. *Let us consider stationary time series x_t*

$$S(\omega) = \sum_h E(x_t, x_{t-h}) e^{-i\omega h} d\omega \tag{19}$$

is called spectral density of the time series x_t

$E(x_t, x_{t-h})$ is symmetric, thus the quantity is real. Using inverse the Fourier transform I can recover autocovariance $E(x_t, x_{t-h}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{+i\omega h} S_\omega$. This is an important result because I decomposed the autocovariances to frequency components (combination of cosine), that is why I call $S(\omega)$ a spectral density.

An extension of definitions for multivariate series is straightforward. For the VAR process I define the spectral representation in the same way:

$$S_X(\omega) = \sum_{h=-\infty}^{\infty} E(X_t, X_{t-h}) e^{-i\omega h} \ . \tag{20}$$

In this case the non-diagonal elements are complex, so $S_X(\omega)$ is a complex matrix. The off-diagonal elements are called cross-spectral densities. Using the inverse Fourier transform of cross spectral density reconstructs the cross-autocovariances of the process,

$$E(X_{i,t}, X_{j,t-h}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{i\omega h} (S_X)_{i,j} d\omega \ . \tag{21}$$

Thus, spectral density gives us alternative representation of VAR process.

Spectral representation of GEFVD

In the previous part I introduced the Fourier transformation and defined the spectral density. In this section, I use the spectral density to construct GEFVD on a frequency band.

I use the fact that for MA representation of one dimensional time series $y_t = \sum_{h=0}^{\infty} \psi_h \epsilon_{t-h}$ the autocorrelation $E(x_t, x_{t-h})$ is equal to $\sum_h \psi_t \psi_h \sigma_\epsilon^2$ and extend this result to MA representation of a vector autoregressive process. Assuming that $x_t = \sum_{h=0}^{\infty} \Psi_h(L)\ \epsilon_{t-h}$

$$E(X_t, X_{t-h}) = \sum_h \Psi \Sigma \Psi \ . \tag{22}$$

Let us define $\Psi(e^{i\omega})$ as the Fourier transform of MA coefficients: $\Psi(e^{i\omega}) = \sum_h e^{-ih\omega} \Psi_h$

The identity (21) gives us representation of the spectral density

$$S_X(\omega) = \sum_{h=-\infty}^{\infty} E(X_t, X_{t-h}) e^{-i\omega h} = \Psi(e^{-i\omega}) \Sigma \Psi^T(e^{+i\omega}) \ , \tag{23}$$

where Σ in the covariance matrix of ε_t

Following the approach of Baruník and Křehlík (2018), who built on Stiassny (1996), I define the generalised causation spectrum as follows

Definition 5. *The generalised causation spectrum over frequencies ω is defined as*

$$(\theta(\omega))_{ij} = \frac{\Sigma_{ii}^{-1} \mid (\Psi(e^{i\omega})\Sigma)_{ij}) \mid^2}{(\Psi(e^{i\omega})\Sigma\Psi(e^{i\omega})_{jj}} \, , \tag{24}$$

where $\Psi(\omega)$ is Fourier transform of MA reprezetation. $\Sigma_{i,i}$ is i-th diagonal element of innovation covariance matrix Σ

The definition of the generalised causation spectrum reflects the definition of GFEVD. The generalised causation spectrum tells us what part of variance in i-th variable is associated with variance in j-th variable within the given frequency ω.

To get the decomposition of GFEVD Baruník and Křehlík (2018) proposed to define the weighting function that represents the share of variance with frequency ω on the total variance of the system.

Definition 6. *the weighting function for j-th variable $\Gamma_j(\omega)$ is defined as fraction*

$$\Gamma_j(\omega) = \frac{\Psi(e^{i\omega})\Sigma\Psi(e^{i\omega})_{jj}}{\frac{1}{2\pi}\int_{-\pi}^{\pi} \Psi(e^{it})\Sigma\Psi(e^{it})_{jj} dt} \, . \tag{25}$$

The following theorem shows the relationship between GFEVD proposed by Diebold and Yilmaz (2012) and the generalised causation spectrum defined above.

Theorem 1. *(Baruník and Křehlík (2018)) Assume that X_t is wide-sense stationary process with $MA(\infty)$ representation and $\sigma_{jj}^{-1} \sum_{h=0}^{\infty} \mid (\Psi_h\Sigma)_{i,j} \mid < \infty$. Than*

$$(\theta_\infty)_{ij} = \frac{1}{2\pi} \int_{-\pi}^{\pi} (\Upsilon(\omega))_{ij} \Gamma_i(\omega) d\omega \, , \tag{26}$$

where $(\Theta_\infty)_{ij}$ is the directional connectedness defined from Diebold and Yilmaz (2012) for $H \to \infty$.

The theorem reflexes how the GFEVD proposed by Diebold and Yilmaz can be reconstructed from the causation spectrum using weighting function that were defined earlier. By integrating the general causation spectrum over all possible frequencies I get the general forecast error variance decomposition, which implies that $(\Theta_H)_{ij}$ for $H \to \infty$ is an measure aggregated over all possible frequencies.

Naturally, rather than in a precise frequency, we are interested GFEVD on frequencies within a certain interval (for example frequencies that represents shocks with the persistence between one week and one month). For this reason Baruník and Křehlík (2018) defined the GFEVD on the frequency band $d = (a, b)$ where frequency band is some interval which covers a part of spectrum of frequencies. I can for example think about $d = (-\pi/5, \pi/5)$ which covers low frequencies with a period longer than 10 days .

Definition 7. *GFEVD on the frequency band $d = (a, b), -\pi < a < b < \pi$ is defined as*

$$((\theta)_d)_{i,j} = \frac{1}{2\pi} \int_a^b (\Upsilon(\omega))_{ij} \Gamma_i(\omega) d\omega \tag{27}$$

For any partition of interval $(-\pi, \pi)$ the sum over GFEVD on the frequency band in equal to the aggregated GFEVD. The following theorem formalize this fact.

Theorem 2. *Let D be a set of disjoint intervals d_s such that $\cup\, d_s = (-\pi, \pi)$, than*

$$(\theta_\infty)_{i,j} = \sum_{d_s \in D} (\theta_{d_s})_{i,j} \, , \tag{28}$$

where $(\theta_\infty)_{i,j}$ is GFEVD proposed by Diebold and Yilmaz (2012).

Derivation can be found in Baruník and Křehlík (2018).

The final step is to define the connectedness measures on a frequency band. Firstly, I standardize the GEFVD on a frequency band and obtain

$$(\tilde{\theta_d})_{i,j} = \frac{(\theta_d)_{i,j}}{\sum_j^N (\theta_d)_{i,j}} \ . \tag{29}$$

The standardized GFEVD $(\tilde{\theta_d})_{i,j}$ on a frequency band now corresponds to the pairwise directional connectedness measure $C^H_{i\leftarrow j} = (\tilde{\theta}_H)_{i,j}$ defined earlier, which implies that I for a given frequency band I can define the connectedness measures in the same way as in Diebold and Yilmaz (2012) using GFEVD on the frequency band. I follow the notation from Krehlik and Barunik (2017) and define the connectedness measures within a frequency band.

Definition 8. *The overall connectedness within the frequency band d = (a,b), $-\pi < a < b < \pi$ is defined as*

$$C^d = \frac{\sum_{i,j i\neq j}^N (\tilde{\theta_d})_{i,j}}{\sum_{i,j} (\tilde{\theta_d})_{i,j}} \ . \tag{30}$$

Similarly, Baruník and Křehlík (2018) define *the within directional connectedness from others to i on the frequency band d* which is the share of variance that accounts to other variables as

$$C^d_{i\leftarrow \cdot} = \sum_{j=1 j\neq i}^N (\tilde{\theta_d})_{i,j} \ , \tag{31}$$

and within directional connectedness to others from i on the frequency band d that reflexes the contribution of i-th asset to the variance of other assets as

$$C^d_{\cdot\leftarrow i} = \sum_{j=1 j\neq i}^N (\tilde{\theta_d})_{j,i} \ . \tag{32}$$

Note that these connectedness measures are restricted on the frequency band. For instance, if I took a frequency band that covers low frequencies and calculated that $C^d = 0.95$. It means that the system is supposed to be highly connected in the long run. However, it does not tell us the importance of the long-run connectedness because it also depends on how persistent the process is. In other words, what share of variance is concentrated on low frequencies resp. on high frequencies.

Short-run and long-run components of total connectedness

As the previous example has shown, to decompose the total connectedness on frequency components one needs to weight the within connectedness by the share of the frequency band d on the total variance of process. For this reason, following definition establish the aggregated directional connectedness.

Definition 9. *Aggregated directional connectedness on frequency band d is defined as*

$$(\tilde{C}^d)_{i,j} = (C^d)_{i,j} * \frac{\sum_k (\theta_d)_{i,k}}{\sum_k (\theta_\infty)_{i,k}} \ , \tag{33}$$

where C^d is the within directional connectedness on the band d, θ_d is the GFEVD on the frequency band d and θ_∞ is the GFEVD aggregated over all frequencies $(-\pi, \pi)$.

Having the aggregated directional connectedness I can construct the aggregated connectedness on the frequency band that shows the share of cross-variance shares within frequency in the band $d = (a, b)$ on the total variance of process.

Definition 10. *The aggregated connectedness on the frequency band $d = (a, b)$ is defined as*

$$\tilde{C}^d = C^d * \tau(d) \;, \tag{34}$$

where $\tau(d) = \dfrac{\sum_{i,j=1}^{N}(\tilde{\theta})_d}{\sum_{i,j=1}^{N}(\tilde{\theta}_\infty)}$ *is a weighting function which indicates the share of variance within the band d on the total variance.*

The aggregated directional connectedness from others to i and aggregated directional connectedness to others from we are defined in a similar way. An important implication of definition of aggregated connectedness on the frequency band d is described in the following statement.

Corollary 1. *(Baruník and Křehlík (2018)) Let D be a set of disjoint intervals d_s such that $\cup\, d_s = (-\pi, \pi)$, than*

$$C_\infty = \sum_{d_s \in D} \tilde{C}^{d_s} \;, \tag{35}$$

where $C_\infty = \lim_{h\to\infty}(C^H)$

The equation says that I can decompose the total connectedness defined in Diebold and Yilmaz (2012) on the connectedness over frequency bands, that covers all possible frequencies. Imagine I define only two frequency bands: a short-term and a long-term one that together cover all possible frequencies. We are now able to find the corresponding short-term resp. long-term aggregated connectedness (aggregated over the high, resp. low frequencies), which are corresponding parts of total connectedness over all frequencies. Suppose that the within connectedness in short-run is 0.9 and in the long-run 0.2 and suppose that weights are 0.1 for short-run and 0.9 for long-run. that means the process has long persistence (but still it need it to be stationary). Then the aggregated connectedness on short-term (resp. long-term) frequencies is 0.09 respectively 0.18 and the total connectedness is 0.27. On the other hand, if I calculate that the short-run within connectedness is 0.1, long-run is 0.9. and weights are 0.9 for short-term and 0.1 for long-term than both the within connectedness and the aggregated connectedness are quite different although the total connectedness which does not distinguish frequencies remains on the same level.

Estimation of connectedness on a frequency band

Having described the theoretical background and measures proposed by Diebold and Yilmaz (2012); Diebold and Yılmaz (2014) and Baruník and Křehlík (2018) the estimation of our model follows. Firstly, I show how to estimate the moving average representation. Having estimated the MA representation one needs to estimate Fourier transform of proces in order to construct connectedness on frequency band.

Assuming VAR(k) proces in covariance stationary and week dependent, I can aproximate $MA(\infty)$ by taking H "sufficiently high" and estimate $MA(H)$ using the recursive formula $\Psi_0 = I$, $\Psi_h = \sum_{j=1}^{max\{k,h\}} \hat{\Phi(j)}(\Psi_{h-1})$ for any $h \in \{1, ..., H\}$. With respect to Baruník and Křehlík (2018) I define estimation of spectrial denstity on a frequnecy band d = (a,b) using discrete approximation of Fourier transform (DFT) as

$$\sum_{\omega \in \{\lfloor \frac{aH}{2\pi} \rfloor, ..., \lfloor \frac{bH}{2\pi} \rfloor\}}^{H} \hat{\Psi}_h(\omega)\hat{\Sigma}\hat{\Psi}_h^T(\omega) \;, \tag{36}$$

where $\hat{\Sigma}$ represents estimated covariance matrix of innovations and

$$\hat{\Psi}(\omega) = \sum_{h=0}^{H} \hat{\Psi}_h e^{-2i\pi\omega/H} \;, \tag{37}$$

for $\omega \in \{0, ..., H-1\}$ so that $e^{-2i\pi\omega/H}$ is symmetrically distributed around the complex unit circle.

The estimation of generalized causation spectrum and weighting function follows. Firstly, I estimate Generalized causation spectrum as

$$(\hat{\Upsilon}(\omega))_{i,j} = \frac{\hat{\Sigma}^{-1}_{j,j}((\hat{\Psi}(\omega)\hat{\Sigma})_{i,j})^2}{(\hat{\Psi}(\omega)\hat{\Sigma}\hat{\Psi}(\omega))_{i,i}} \tag{38}$$

and weighting function as

$$\hat{\Gamma}_j(\omega) = \frac{(\hat{\Psi}(\omega)\hat{\Sigma}\hat{\Psi}^T(\omega))_{j,j}}{(\sum_{\omega\in\{\lfloor\frac{aH}{2\pi}\rfloor,...,\lfloor\frac{bH}{2\pi}\rfloor\}} \hat{\Psi}_h(\omega)\hat{\Sigma}\hat{\Psi}^T_h(\omega))_{j,j}} \, . \tag{39}$$

By plugging $\hat{\Gamma}_j(\omega)$ and $(\hat{\Upsilon}(\omega))_{i,j}$ into definition of GFEVD on a frequncy band (a, b) I obtain

$$(\hat{\theta}_d)_{i,j} = \sum_{\omega\in\{\lfloor\frac{aH}{2\pi}\rfloor,...,\lfloor\frac{bH}{2\pi}\rfloor\}} (\hat{\Upsilon}(\omega))_{i,j} \, \hat{\Gamma}_i(\omega) \, . \tag{40}$$

Finally, I can construct the estimation of within and aggregated frequency connectedness on the frequency band d = (a,b) by plugging $(\hat{\theta}_d)_{i,j}$ into the definitions.

4 Data

In our analysis I include seven commodities traded on Chicago Board Options Exchange (CBOE) that represents different sectors of the commodity market - agricultural, energy, precious and industrial metals, fibres and stock market index S& P 500 representing the US Equity. In particular, I use futures prices of Crude Oil (CL), Natural Gas (NG) to represents the trade with energies, Gold (GC) and Silver (SV) to represent precious metals, Copper (HG) as proxy for industrial metals, Corn (CN) that represents the agricultural sector and Cotton (CT) represents fibres. I argue that the selected commodities can be viewed as a proxy for these sectors, which is supported by empirical study of Diebold et al. (2017) who shows that commodities are mostly grouped by sectors, with exemption of some exotic commodities. I decided to include two proxy for energy sector, because Oil volatility and Natural Gas volatility may shows different behaviour. I also included two precious metals, silver and gold because gold has an exceptional position among all commodities.

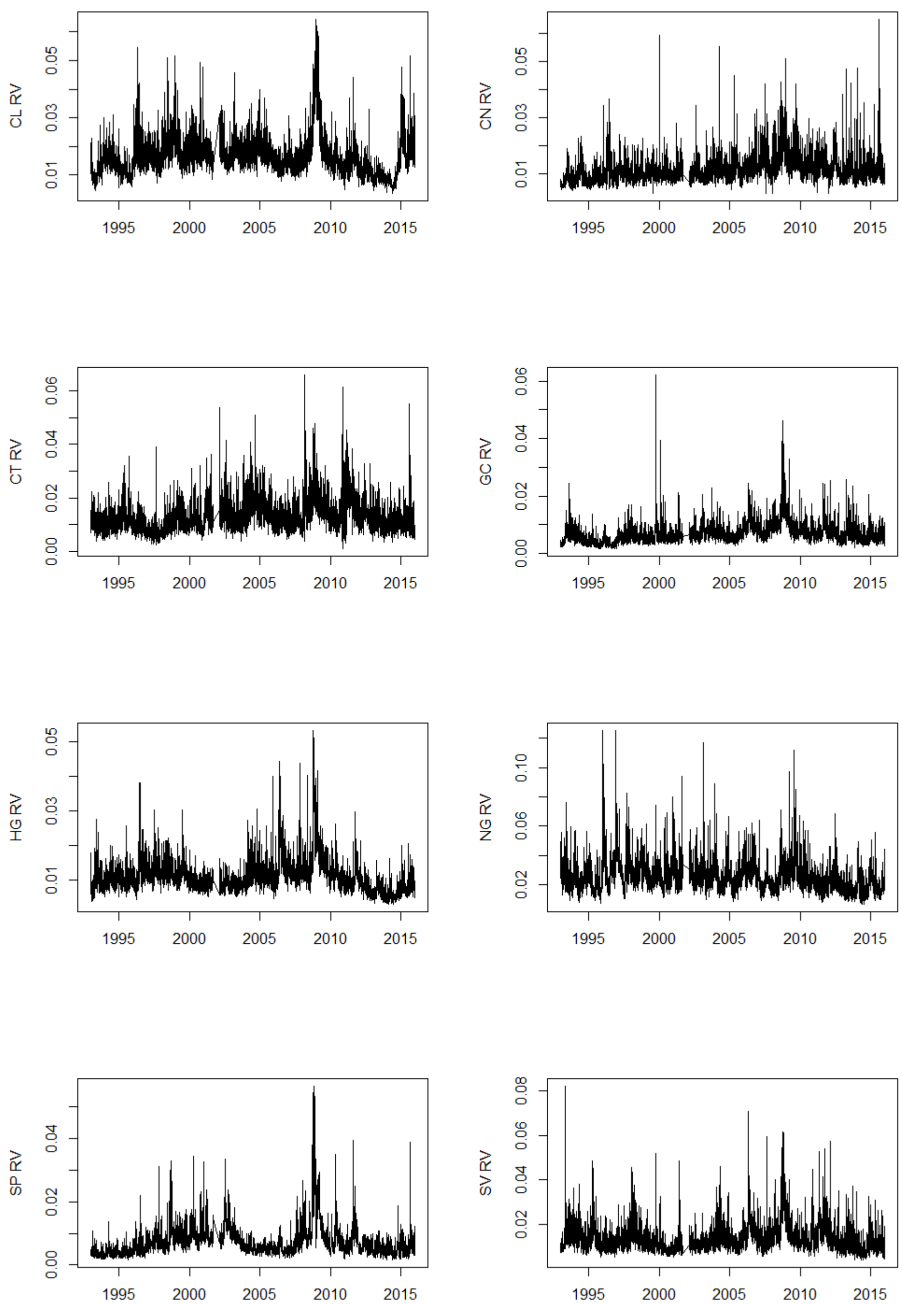
I use extracted 5-minute log-returns from high-frequency data provided by Tick Data Inc. The dataset is adjusted so that days with low trading activity, weekends and holidays are excluded. As we are interested in volatility transmission mechanism, 5-minute returns were used to estimate the realized variance of assets as a sum of squared intra-day returns. I compute realized volatility as the square root of the estimated daily variance, that is commonly used in literature as the ex-post measure of uncertainty on a market. Instead of the realized volatility range-based estimates of volatility can be used. Range-based estimates are computed from opening and closing prices and intra-day minima and maxima, which are available for free arguing that range-based estimates gives almost similar results as the realized volatilities. However, Liu et al. (2015) argue that the realised volatility has better performance than any other conventional measures of volatility.

The data covers the period from January 4 1993 to December 31 2015, which allows us to analyse both pre and post crisis periods. However, it is important to note that in a period after Sep 11 2001, there is lack of observations as the volume of trade decreased sharply. A small number of observations may lead to bias in the estimation, so the period from 11 September 2001 to 11 February 2002 was excluded from our data sample and I split our the sample into two parts - before 9/11/2001 and after 2/11/2002.

Following the approach of Krehlik and Barunik (2017), I compute natural log of the realized volatility in order to get data closer to normal distribution for the VAR model and smooth the connectedness measure.Diebold et al. (2017) argue that for the connectedness measures the logarithms of realized volatilities are more suitable for measuring connectedness using VAR model as daily realized volatilities tends to be distributed asymmetrically while the natural logarithm of RV has approximately normal distribution.

Our data shows significant increase in the volatility of Energies, Metals and S&P 500 during the period 2008-2009 (Figure 1) Fibres and Agricultural seems to be less effected. Occasional high uncertainty on the markets is less evident from log(RV) although it is still present in the data. The logarithmic transformation of our time series can be found in the appendix as well as the normal Q-Q plot which support the hypothesis that logarithmic transformation of RV is close to normal distribution. While quantiles of log transformation of RV roughly correspond to quantiles of normal distribution, Gaussian Q-Q plot of realised volatility shows the distribution is rightly skewed. Table 2 summarize key features of our data, showing that the highest mean and standard deviation of the realized volatility is reported for Natural Gas, followed by Crude Oil.

Figure 1: Realized volatility of the commodities and the equity index



Source: Author's computations based on the data from TickData Inc.

Table 2: Descriptive statistics of RV

Statistic	N	Mean	St. Dev.	Min	Max	Skewness	Kurtosis
CL	5,322	0.017	0.007	0.004	0.064	1.73	5.45
CN	5,322	0.012	0.005	0.003	0.065	2.28	11.16
CT	5,322	0.013	0.006	0.001	0.066	1.68	5.71
GC	5,322	0.007	0.004	0.002	0.062	2.81	18.53
HG	5,322	0.011	0.005	0.003	0.053	2.31	9.46
NG	5,322	0.025	0.011	0.007	0.126	2.05	8.88
SP	5,322	0.007	0.005	0.002	0.057	3.12	16.62
SV	5,322	0.014	0.006	0.004	0.082	2.58	11.91

Source: Author's computations based on the data from TickData Inc.

5 Results

This part provides an empirical evidence of connections among commodities and the equity market. Firstly, I show the static analysis of spillovers computed in three periods: before the terrorist attack on September 11, in period from 2002 to 2007 and finally after the global financial crisis, excluding several months after the terrorist attack in September 11, 2001. Secondly, I provide analysis of connectedness dynamics that is using a rolling span over 250 days which roughly corresponds to one-year span. This will give us more detailed view on the evolution of connectedness.

Stationarity of our model

In our model, I assume to work with a stationary or at least with local stationary process, thus before I start with the calculation of connectedness, I check whether VAR system does not consist of a unit root process, which would probably bias the OLS estimation of VAR model. For this purpose, I deployed the Augmented Dickey-Fuller test (ADF). The null hypothesis of the ADF test in that unit root is present in the logarithmic transform of the realized variances. The alternative is that the process is stationary. The following table shows values of ADF statistics for each variable and P-values of the test. The lower ADF statistics is the stronger is rejection of the null hypothesis. The critical value of test on level 0.05 is -2.86. Thus I conclude that our data do not involve unit root at the 0.05 level. P-value is lower than 0.01 for all commodities, thus I can reject the null hypothesis even at 1 % level (Figure 3).

Table 3: ADF test of unit root in log realized volatilities with 17 lags.

	ADF Stat.	P-value
CL	-5.15	0.01*
CN	-8.02	0.01*
CT	-5.97	0.01*
GC	-6.08	0.01*
HG	-5.64	0.01*
NG	-7.89	0.01*
SP	-5.83	0.01*
SV	-6.66	0.01*

*P-value is lower that 0.01

Static short-run and long-run connectedness

Here I show results of connectedness computed on three data samples. The first dataset spans from 4 January 1993 to 10 September 2001. The second covers period from 11 February 2002 to 22 February 2007, before the signs of sub-prime mortgage crisis in 2007 were noticed by markets [3] [4] [5]. Last part of our data spans from 23 February 2007 to 31 December 2015. I obtain three connectedness matrices for each time period. The first table shows volatility connectedness between assets regardless to persistence of volatility spillovers, in other words it is volatility connectedness aggregated over all possible frequencies. The second table describes pairwise connectedness on a the frequency band $(\pi, \pi/10)$ that corresponds to volatility spillovers with a persistence shorter than 10 days, which I call the short-run connectedness table. The third table shows long-run spillovers among the selected commodities and equity, aggregated over low frequencies. All tables, which represents pairwise directional spillovers among assets, can be found in the appendix. Here I focus on the aggregated measure of system connectedness as well as on the volatility transmission from commodities and S&P 500 to the commodity and equity market and identify net sources of volatility to the market. Functions for calculation of spillovers are built in R. [6] I use VAR of order 2 selected based on AIC criterion in Baruník and Křehlík (2018) and Krehlik and Barunik (2017).

System-wide connectedness

System-wide connectedness went through significant changes during the studied period. In the period from 1993 to 2001 the total connectedness of system over all degrees of persistence was roughly 16 %, while in 2001-2007 the total connectedness is estimated to be 22 %. Since 2007 the connectedness of system rose even more, to 41 %, implying that approximately 41 percent of volatility of commodities is caused by not-own shocks, which is twice as much as in the pre-crisis period. I will further investigate this effect using the rolling window estimation of connectedness.

Looking now at frequency components, in the years 1993-2001 the short run overall connectedness was roughly 6 % , representing approximately 40 percent of the total volatility spillovers and long-run spillovers, with the persistence longer than 10 days, accounts the remaining 60 percent. During the period 2002-2007 short run volatility connectedness rose to 9,5 % therefore short-run connectedness increased in both ways - in the absolute value about 3.5% and as the percentage share on overall connectedness it increased to 43 percent. Thus markets started to process informations more quickly than before. Long-run absolute connectedness was approximately 12%. In the period after the crisis, long-run volatility connectedness grew to more than 0.3 and accounted for 3/4 of total connectedness of the system. Thus the period after the global financial crises is characterized by high volatility spillovers that creates long persistent linkages in volatility of the system.

Once more, I will look at the connectedness of the system now excluding the equity index S&P 500. The total connectedness in the years 1993-2001 is estimated to be 12 %, which is about 3 % less than in the full sample. Approximately one half of connections had persistence shorter than two weeks. In contrast, the total connectedness without equity index in the data sample in the years 2001-2007 is 22 %, which is the same number as in full sample in 2001-2007. In the years 2007-2015 the total connectedness commodities is roughly 38 %. I can conclude that the total connectedness of commodities seems to be relatively robust to exclusion of equity from our data sample.

I shall continue with an analysis of the directional connectedness that should shed light on the sources of volatility as well as on the effect of financialization of commodities

[3] financialconnectedness.org/

[4] https://www.census.gov/en.html

[5] money.cnn.com/2007/02/26/news/economy/greenspan/index.htm

[6] results are computed using R package frequencyConnectedness. The package is available at https://github.com/tomaskrehlik/frequencyConnectedness

that occurred in recent years.

Directional connectedness

Here I investigate the directional connections in volatility among selected commodities and S&P 500. I remind that TO connectedness of i-th asset represents the share of volatility that is explained by i-th asset and can be obtained as the sum of off-diagonal elements in i-th column while FROM connectedness in the sum of rows of connectedness table and represents what share of uncertainty in i-th variable is caused by a shock in other variables from the system. Most importantly, the NET connectedness, which is the difference between TO and FROM, shows if the asset is a net contributor to volatility of market and therefore a source of uncertainty.

Firstly, let us consider the period before 2001 when the connectedness of system was relatively low. TO, FROM and NET connectedness in the years 1993-2001, indicate that volatility of Gold, Silver end Equity accounts for large part of volatility linkages in the market(Table 4), which is mainly due to high volatility transmission from stock market to Oil as well as from Gold to Silver and other way around [7]. Energies such as Crude Oil and Natural Gas were net receivers of volatility as well as Corn and Copper, that represents agriculture and industrial metals. The data shows that in the period from 1993 to 2001 S&P 500 was the main source of the uncertainty that spills to commodities. Spillovers from SP accounted for 3.8 % of volatility in commodities. NET connectedness shows that spillovers from SP to Commodities excess by 1,9 % spillovers from commodities to stocks. Decomposed on short-run and long-run parts, net contribution to variance in short-run was approximately 0.4 % while in the long-run it was 1.5 %. In detail, most sensitive to shocks in S&P 500 were Crude Oil and Gold. About 12 % of variance in Crude Oil was received from the stock market from which over 90 % was high persistent. In case of Gold it was roughly 6%, with higher share of long persistent shocks.

Generally, in the period after 11 September, 2001 the commodity market became more interconnected and less dependent of stocks (Table 5). Especially metals became more integrated. One fifth of volatility in Silver accounts to shocks to Gold and symmetrically 20 percent of volatility in gold transmits from Silver. Silver is also responsible for 17 percent of volatility in Copper (Table 10). For precious metals both short-run and long-run connections have similar importance. Copper is more connected with others on long-term horizon (Tables 11,12).

On the other hand, connectedness of the equity and commodity market decreased to almost a half compared to the previous period, although S&P 500 is still net giver of volatility. Volatility that transmitted from commodities to equity has mostly long persistence while a half of the volatility that spills from stock market to commodities has short-run nature.

After the global financial crisis directional changed dramatically. As the total connectedness doubled compared to the years 2001-2007, with the exception of Natural Gas, spillovers from all assets increased (Table6). Volatility of Crude Oil now accounts for approximately 6 % of volatility of the commodities and stocks. Uncertainty of Oil prices were affected mainly by shocks in the stock market and Copper and on the other hand, uncertainty about oil prices propagates to Copper, S&P and Natural gas (Table13). Connections among oil, industrial metals and stocks is new phenomenon. While in 2001-2007 less than 1 % of volatility spills from oil to S&P in 2007-2015 it was about 9%. Connectedness of copper and oil rose more than six times. All results show higher integration of energies and other sectors, including stocks. Spillovers from crude oil had mainly long-term persistence.

Our model shows that the effect of the stock market on the commodity market rose on its importance, as the shocks in stock prices now accounts for 7% of variance in commodities on average, where roundly 6 % has long persistence (Table 6). Net contribution of S&P rose to 2.7 %. Volatility from stock market transmitted mainly to metals and

[7] Pairwise connectedness tables are included in appendix.

Crude Oil. Surprisingly, importance of Natural Gas as the source of volatility decreased in 2001-2007 as NG accounted for 1,6 % of variance in other assets while in the period after crisis it was about 0.7%.

Table 4: TO,FROM and NET connectedness over the period 1993-2001

Total	CL	CN	CT	GC	HG	NG	SP	SV
TO	1.691	0.978	0.480	3.861	1.207	0.556	3.893	3.257
FROM	2.260	1.566	0.674	3.288	1.643	0.961	2.005	3.527
NET	-0.569	-0.588	-0.194	0.573	-0.435	-0.405	1.888	-0.269
Short-run								
TO	0.338	0.245	0.135	2.081	0.434	0.149	0.738	1.844
FROM	0.275	0.289	0.176	1.834	0.650	0.226	0.381	2.134
NET	0.063	−0.043	−0.041	0.247	−0.215	−0.077	0.357	−0.290
Long-run								
TO	1.353	0.733	0.344	1.780	0.773	0.407	3.155	1.413
FROM	1.985	1.277	0.498	1.454	0.993	0.735	1.624	1.393
NET	−0.632	−0.544	−0.154	0.326	−0.220	−0.328	1.531	0.020

Table 5: TO,FROM and NET connectedness over the period 2001-2007

Total	CL	CN	CT	GC	HG	NG	SP	SV
TO	2.298	0.411	0.845	5.341	3.585	1.643	1.972	5.743
FROM	2.422	1.130	1.284	4.942	4.020	1.644	1.383	5.013
NET	-0.123	-0.720	-0.439	0.399	-0.435	-0.001	0.589	0.731
Short-run								
TO	1.371	0.160	0.348	2.527	1.074	0.894	0.875	2.298
FROM	1.414	0.275	0.518	2.483	1.053	1.116	0.431	2.259
NET	-0.043	-0.114	-0.169	0.043	0.021	-0.222	0.445	0.040
Long-run								
TO	0.927	0.250	0.497	2.814	2.511	0.749	1.097	3.445
FROM	1.008	0.856	0.766	2.459	2.967	0.528	0.952	2.754
NET	-0.080	-0.606	-0.270	0.355	-0.456	0.221	0.145	0.691

Table 6: TO,FROM and NET connectedness over the period 2007-2015

Total	CL	CN	CT	GC	HG	NG	SP	SV
TO	5.670	2.073	1.834	7.180	9.085	0.772	7.289	7.332
FROM	5.513	3.522	3.435	7.333	6.451	3.124	4.546	7.311
NET	0.157	-1.449	-1.600	-0.153	2.634	-2.352	2.743	0.020
Short-run								
TO	1.203	0.461	0.393	2.552	1.636	0.259	1.344	2.468
FROM	0.831	0.780	0.686	2.751	1.125	0.633	0.875	2.633
NET	0.372	-0.318	-0.293	-0.200	0.511	-0.375	0.468	-0.165
Long-run								
TO	4.468	1.612	1.441	4.628	7.449	0.514	5.946	4.864
FROM	4.682	2.742	2.749	4.582	5.326	2.491	3.671	4.678
NET	-0.215	-1.130	-1.307	0.046	2.123	-1.977	2.275	0.185

Dynamic connectedness

In this subsection I discuss the evolution of connectedness over the time paying special attention to development after 2007. Due to the lack of observations in the period after 11 September, 2001 our I split the data sample into two parts. One covers the period before the terrorist attack on 11 September, 2001 starting from 4 January, 1993, the second covers the period from 11 February, 2002 to 31 December, 2015. Using the rolling sample over 250, I show how connectedness varies over time. As in the previous part, I use the vector autoregressive model with two lags and prediction horizon H = 100, to approximate infinite prediction horizon.

Dynamic system-wide connectedness

At first glance, connectedness in the first period before the fall of the Twins was relatively low and stable compared with the second period 2002-2015, when connectedness vary a lot and increases in the turbulent times of global financial crisis and European debt crisis (Figure 2).

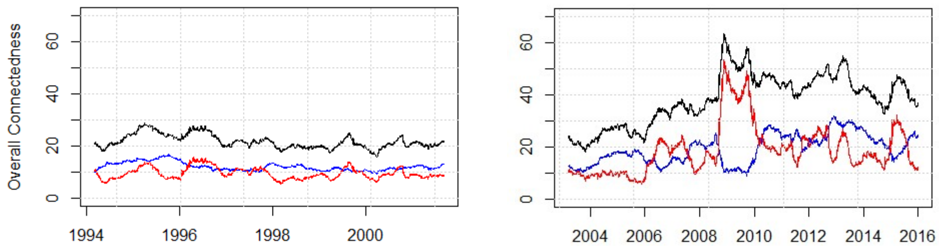
The total connectedness of system remained under 30 percent, until an increase in long term connectedness from 6 % in November 2005 to 21 % in the mid 2006. The increase in long persistent connections was partially compensated by short-run connectedness that dropped from 17 % in November 2005 to 11 % in July 2006. This event reflexes the announcement of changes in FED interest rate. In July 2006 Fed announced to higher the U.S. interest rate to slow down the overheated U.S. economy[8], which is likely to explain increase of volatility transmission among the commodity and the equity market, as changes in interest rate effects prices of commodities. Beside S&P, precious metals were in this case the sources of increased volatility connectedness. It is not surprising that volatility propagates through gold and silver as they are historically more connected with the money market than other commodities, due to the fact that gold and silver were historically considered as currencies and they were used by investors as hedge against inflation. Looking at the realized volatilities, in summer 2006 both Gold and especially Silver report high RV, thus there were high uncertainty about prices of precious metals.

In the period from July 2006 to the end of august 2008 the total connectedness remained within the range from 30 percent to 40 percent. However, long-term connectedness went down by more than half on 10 % in November 2007, implying that from August 2007 to August 2008 markets created mainly linkages with a persistence shorter than two weeks, thus informations were processed by markets relatively quickly.

This have completely changed after the announcement of bankruptcy of Lehman Brothers in September 2008. Up to this moment, it was believed that if large banks were "to big to fail" and that the U.S. authorities would interfere. This step was followed by large volatility spillovers that propagates to other markets as well. Shortly after the fall of Lehman Brothers crisis spills to the world financial system affecting various banks in many countries. The connectedness in volatility of commodities and equity went up to more than 60 %. Most importantly, our model shows the change in the persistence of connections. During the financial crisis, most of volatility spillovers accounted to connections on low frequencies. Long-run connectedness rose by more than four times, whereas short-run decreased by a half. One possible interpretation is, that the believe in the whole financial system was threatened, which lead to long persistent uncertainty on commodity market. Total connectedness in volatility reached its peak in the end of November 2008. I calculate connectedness on one year rolling span, thus until November 2009 connectedness remained on high level. It seems that none of events that happened on financial market spills to commodity market more than the crisis in late 2008.

[8]https://www.nytimes.com/interactive/2015/12/11/business/economy/fed-interest-rates-history.html

Figure 2: Total connectedness of the system. Black line depicts overall connectedness. Red line and Blue line depicts long-run connectedness and short-run connectedness respectively



Dynamic volatility spillovers between the stock and the commodity market

In this subsection I focus on directional links between commodities and equity. At first glance, the data provides evidence of low connectedness of equity and commodity 1993-2001 which contrast with the second period. Directional spillovers between the commodity market and stock market show large contribution of stock prices volatility to uncertainty on the commodity market during the recent global financial crisis 2008-2009 and the European debt crisis 2011-2013. Looking at frequency components of spillovers from S&P there is a significant difference between low and high frequency volatility connectedness. The short-run NET connectedness was close to zero during the whole period. On the other hand, Long-run connectedness shows significant volatility transmission from equity to the commodity market. During the global financial crisis long-run spillovers rose to more than 15 %.

The directional volatility connectedness from commodities to equity doubled in 2008. Since than it remained close to 5 %. Short-run and long-run spillovers from the commodity market accounts for approximately the same share of total connectedness during the period 2002-2015 ,with the exception of the turbulent years 2008-2009 when linkages were mostly high persistent.

Figure 4 reports net pairwise spillovers from SP to commodities, indicating that, with the exception of natural gas, all commodities receive more volatility from the equity than they give. In line with the previous results, connections between equity and commodities create mainly linkages with higher persistence than two weeks. Figure 3 shows that both spillovers from the equity markets to commodities and the spillovers from the commodity market to the SP increased during the downturn in the end of 2008, but spillovers from SP dominated the other way around in all pairwise connections.

The largest net volatility transmission from SP was to Oil, which reached 4 % in 2008-2009. Net effect of SP on Oil was for most of the period positive, with the exception of late 2014 when Oil prices collapsed, which lead 1 % volatility transmission from Oil to S&P. The second most effected commodity from the sample was Copper, which received more than 3% of volatility from SP in the years 2008-2009.

Short-run net volatility transmission from SP fluctuated between -0.5 and -0.5 in case of all commodities, showing that only the dynamics of long term linkages matters.

Figure 3: Volatility spillovers between the commodity and stock market in the period 2002-2015. Black line shows overall TO, FROM and NET connectedness, Red and blue line shows long-run respectively short-run connectedness.

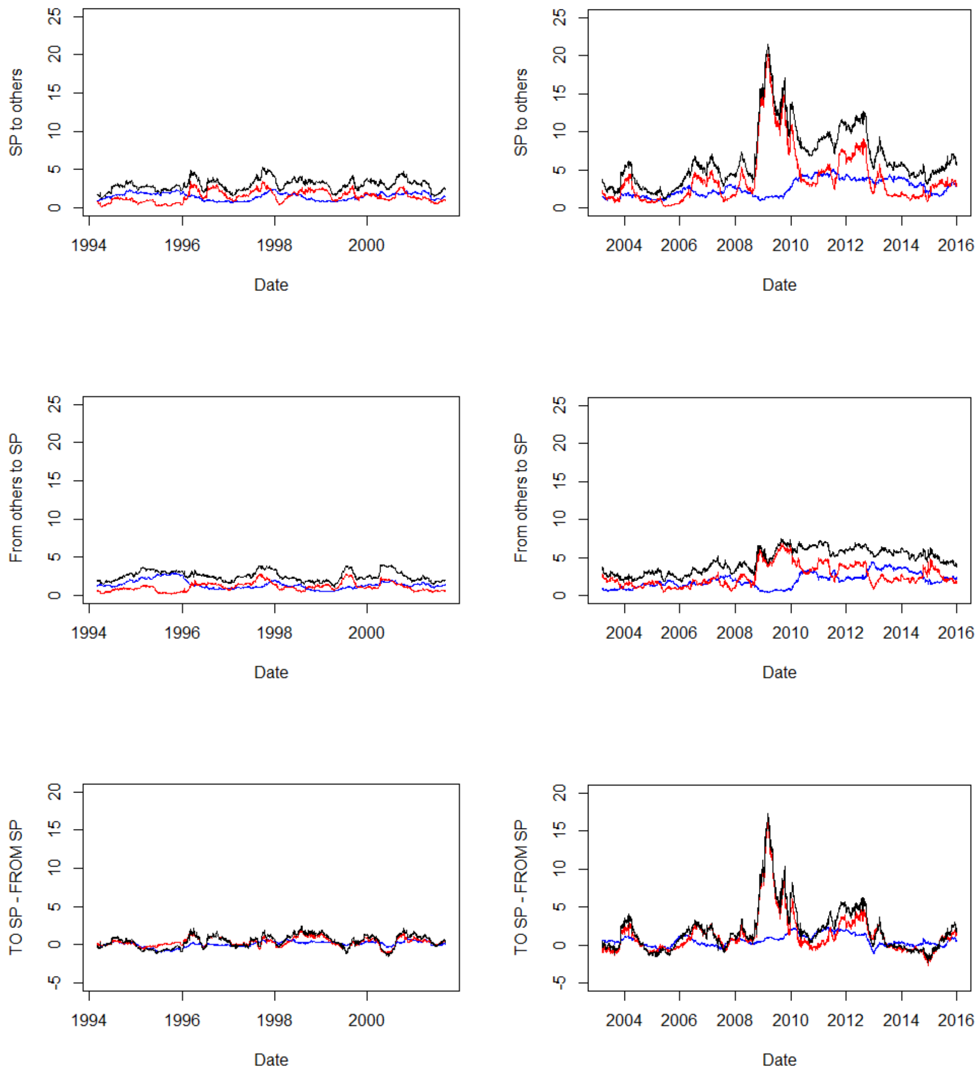
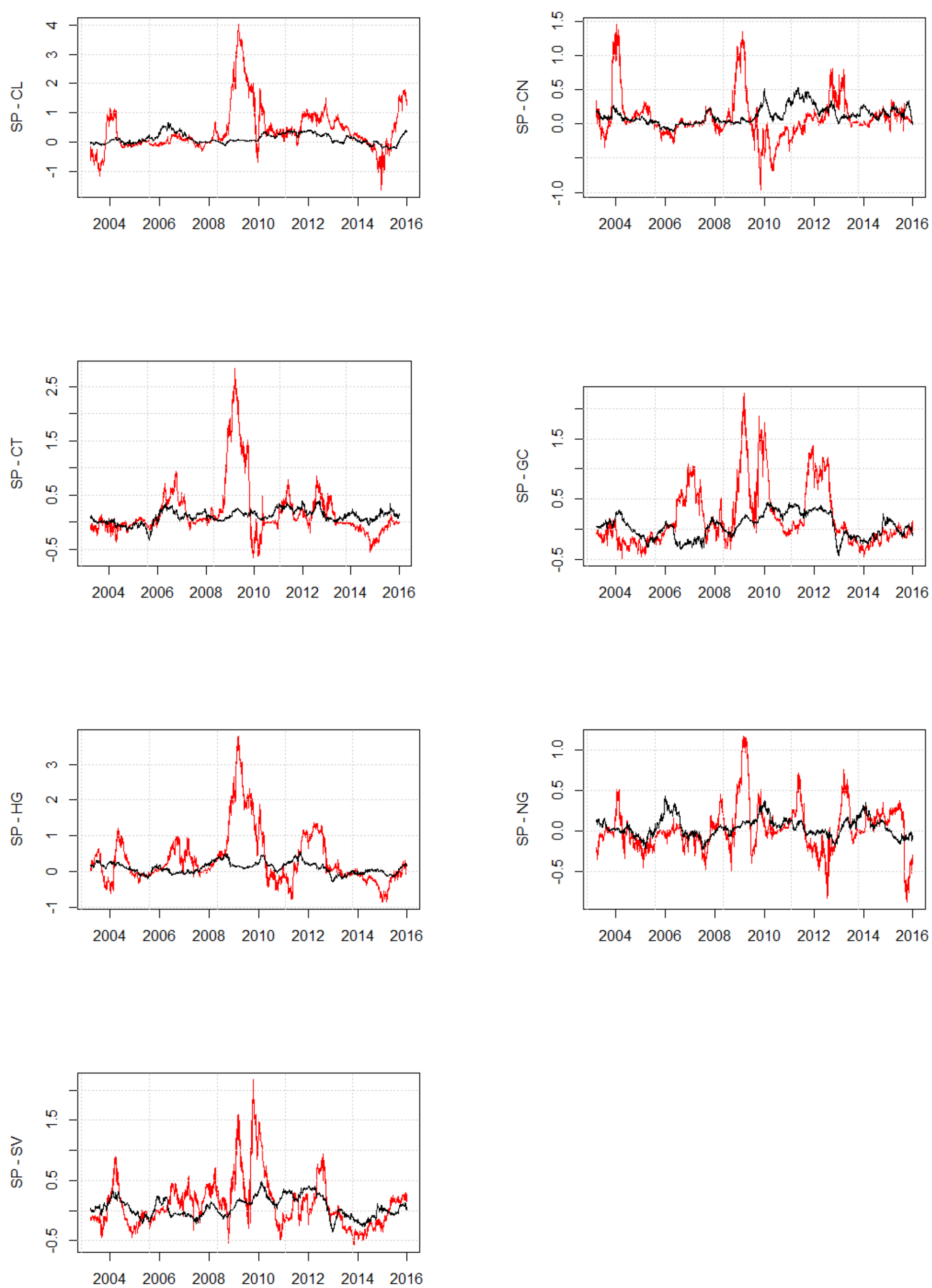


Figure 4: Net volatility spillovers from S&P 500 to commodities in the years 2002-2015. Red line presents connections with higher persistence than 10 days, black line presents linkages with persistence lower that 10 days.



Directional volatility spillovers from commodities to markets

This subsection is devoted to the dynamics of spillovers among commodities. As I can not report all pairwise directional connectedness, I rely on TO and FROM aggregated directional spillovers, that report the share of variance they contribute to volatility of others, respectively the share of variance that is received form others. Finally, NET directional spillovers report the net contribution to volatility of market which is probably the most interesting quantity. The dynamics connectedness aggregated over all frequencies is well documented in Diebold et al. (2017), thus I report only the frequency components. The results are summarized in Figures 5,6 and 7.

Considering the period before 11 September, 2001 volatility spillovers from Oil to markets were limited as both short-term and long-term linkages remained under 2 %. Spillovers rose on importance in the years 2002-2015. Sharp drop in prices of CL in 2008 was accompanied by long persistent volatility spillovers from oil to others, that reached 5% in the end of 2008. When the oil prices recovered in 2009, long-term connectedness reached more than 10 %. In 2011-2014 when prices of oil remained relatively high fluctuating between 80-110 dollars per barrel, the importance of short-term volatility transmission rose and both long and short-term To connectedness accounted for approximately the same portion of spillovers. When prices of CL collapsed in the end of 2014, the TO connectedness reached unprecedential level of long persistent volatility spillovers as 15 % of oil volatility propagates to the commodity and the equity markets.

The dynamics of natural gas differs. Spillovers from NG were limited for the whole period. Long term connectedness fluctuated between 0.5 and 4% in both periods 1993-2001 and 2002-2015. Short-term fluctuated between 0.5 and 2 %. NET connectedness shows that Natural Gas was net receiver of volatility in 2014 when Oil prices plunged.

Unlike other commodities in the sample Gold and silver reports higher importance of short run connections. Our hypothesis is that it results from high short run connectedness between gold and silver. This is in line with results from static analysis, that reveal high integration of precious metals as well as high connection among precious and industrial metals.

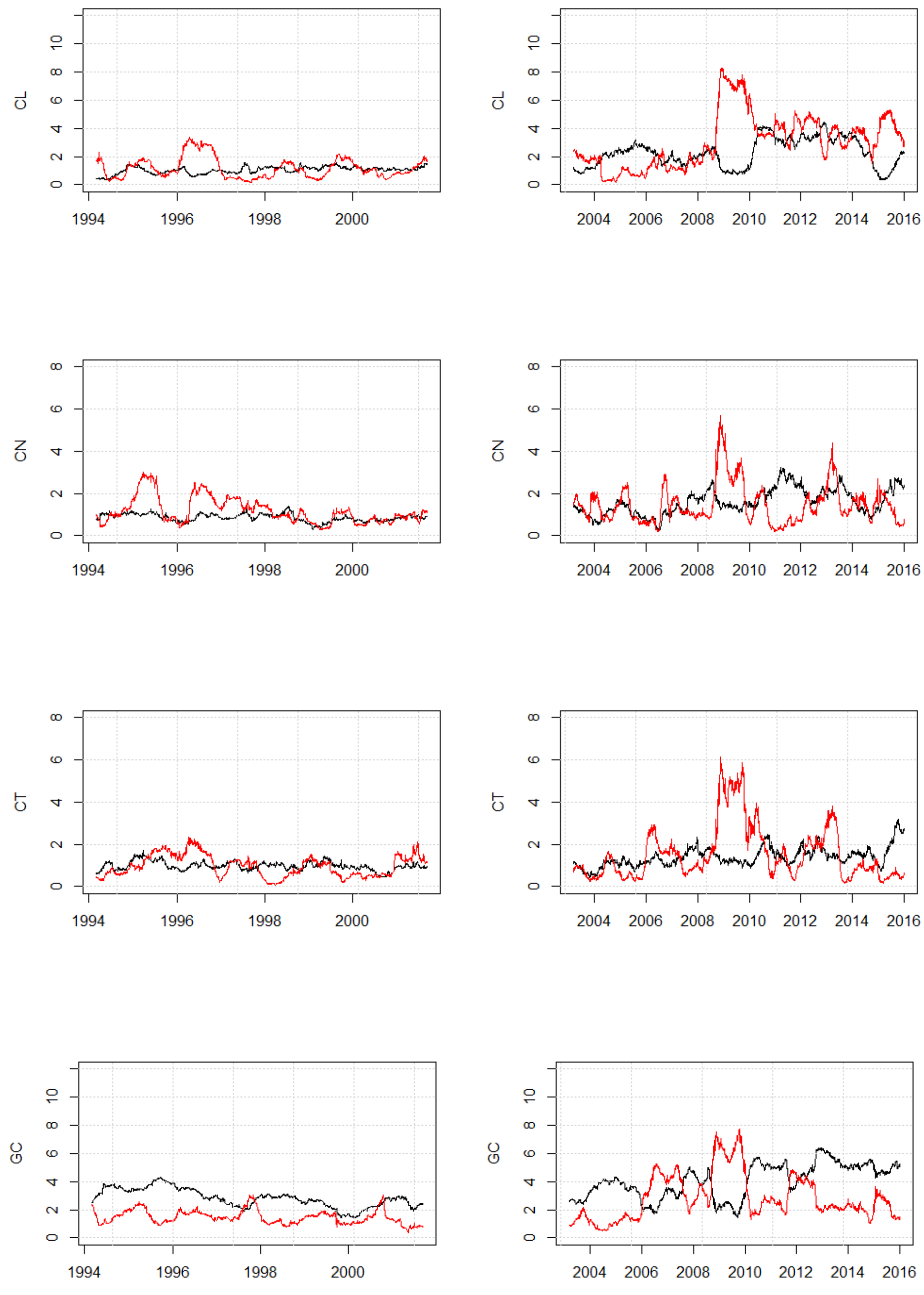
NET contribution of gold to volatility of others fluctuates between positive and negative values and it is hard to find any pattern in net connectedness of gold. Long term spillovers to gold as well as long persistent spillovers from gold to markets reached the highest level in the years 2008-2009.

Spillovers from silver to the equity and commodity markets and from markets to silver follows the same pattern, which can be expected as silver and gold are highly interconnected. As I showed in the static analysis of spillovers, industrials and precious metal tends to be highly connected. Thus it is not surprising that the development of directional FROM and TO spillovers of copper shows similar dynamics to gold and copper. Compared to gold and silver, spillovers form copper to others were more limited, especially in the period 1993-2001 when spillovers from copper were almost negligible.

Corn that represents U.S. agricultural market received around 5% of volatility from other markets. When the financial crisis spread, corn was net receiver of long persistent volatility. In the end of 2009 Corn contributed significantly to volatility of other commodities market, which accompanied the drop in prices of agricultural commodities.

Cotton was in the period 1993-2001 one of the most independent commodities in the sample with respect to transmission of uncertainty. However, our data shows significant contribution of other markets to volatility of cotton, with long persistent character. NET connectedness of cotton with markets shows that volatility of cotton is affected by other commodities rather than the other way around. During the financial crisis 2008-2009 fig. 6. reports net volatility contribution of other assets to volatility of cotton with long persistence.

Figure 5: Directional spillovers FROM others. Red line presents directional FROM volatility spillovers with persistence longer than two weeks. Black line shows FROM connectedness with persistence shorter than two weeks.



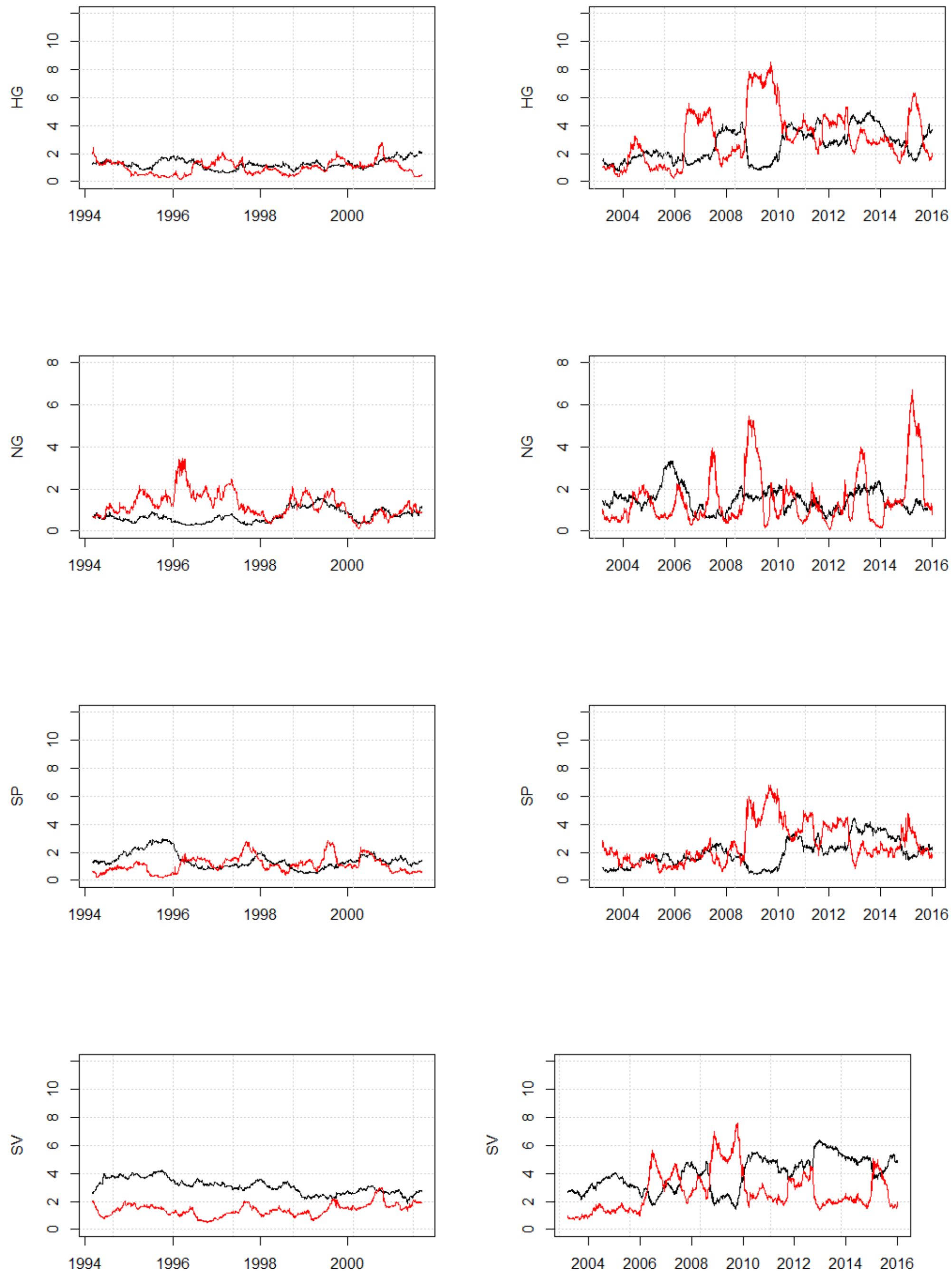

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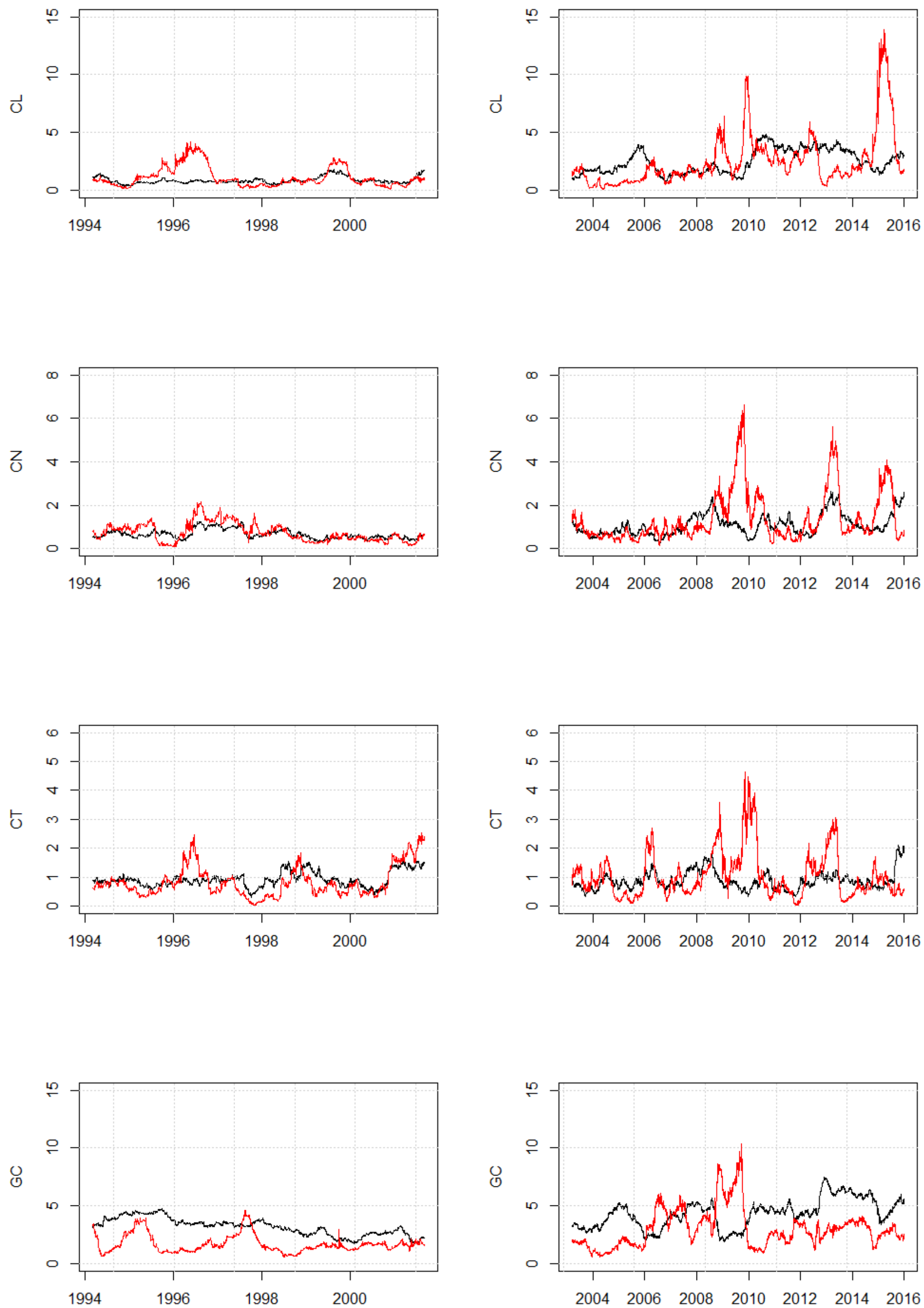
Figure 6: Directional spillovers TO others. Red line presents directional TO volatility spillovers with persistence longer than two weeks. Black line shows TO connectedness with persistence shorter than two weeks.



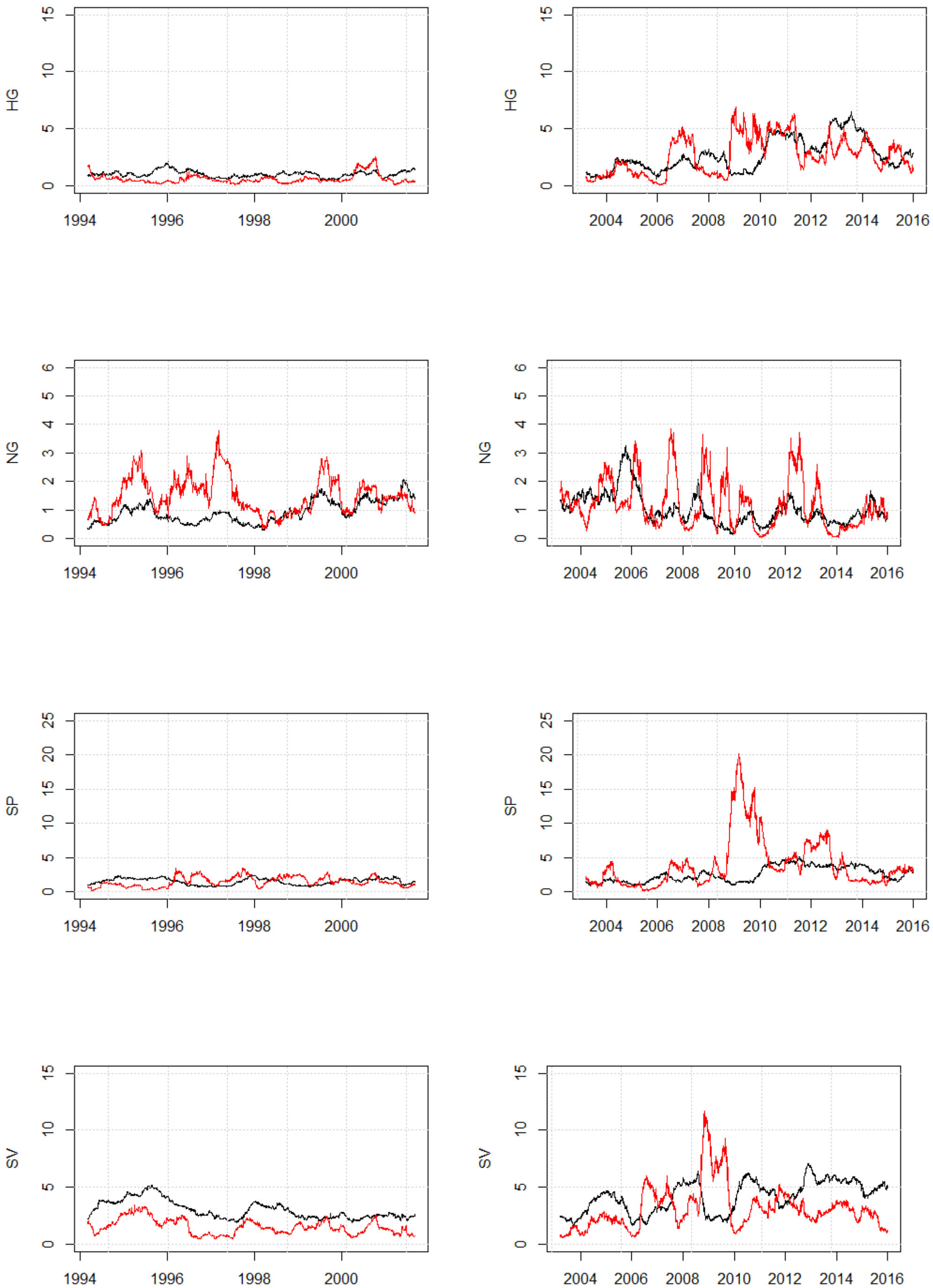

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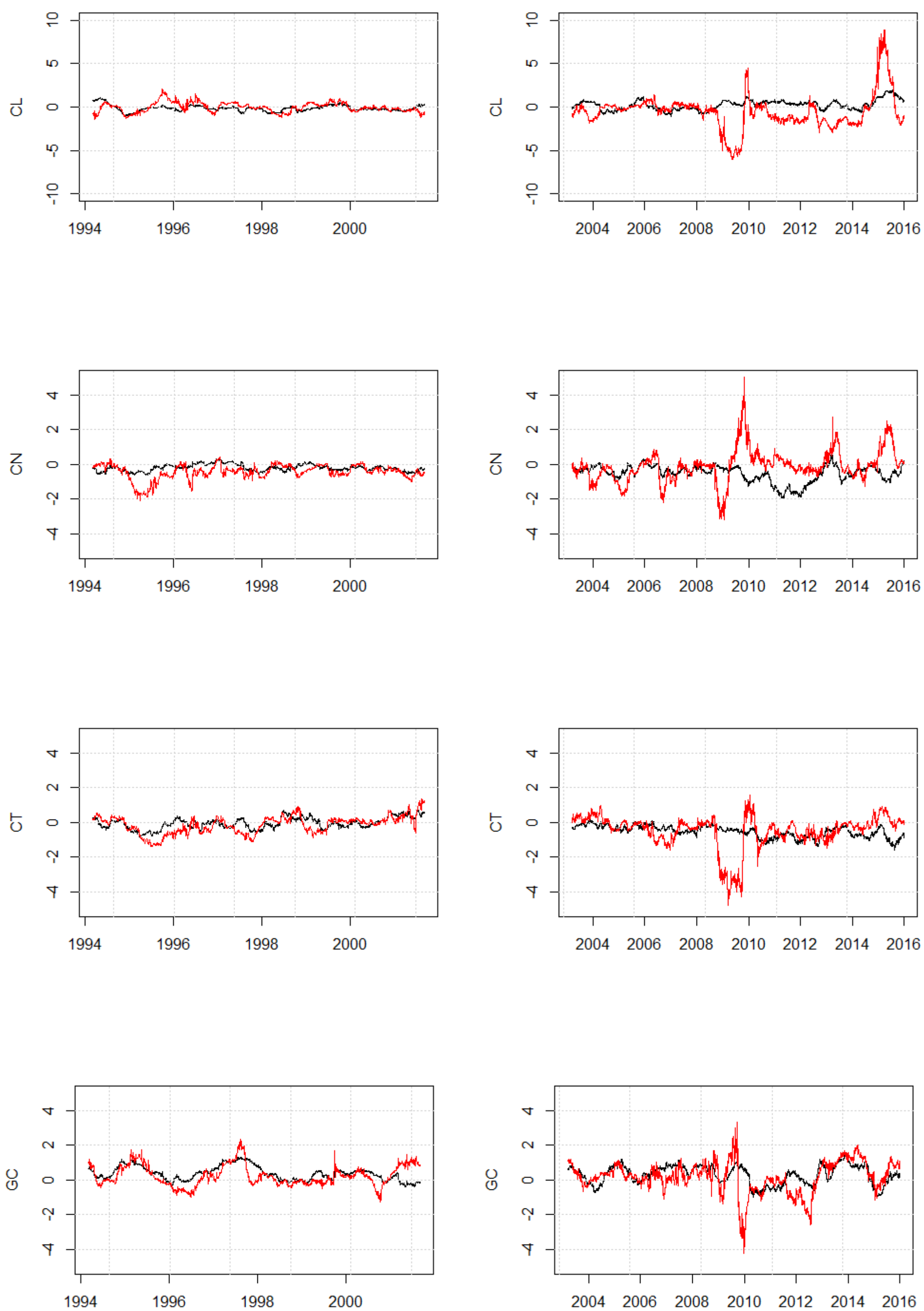
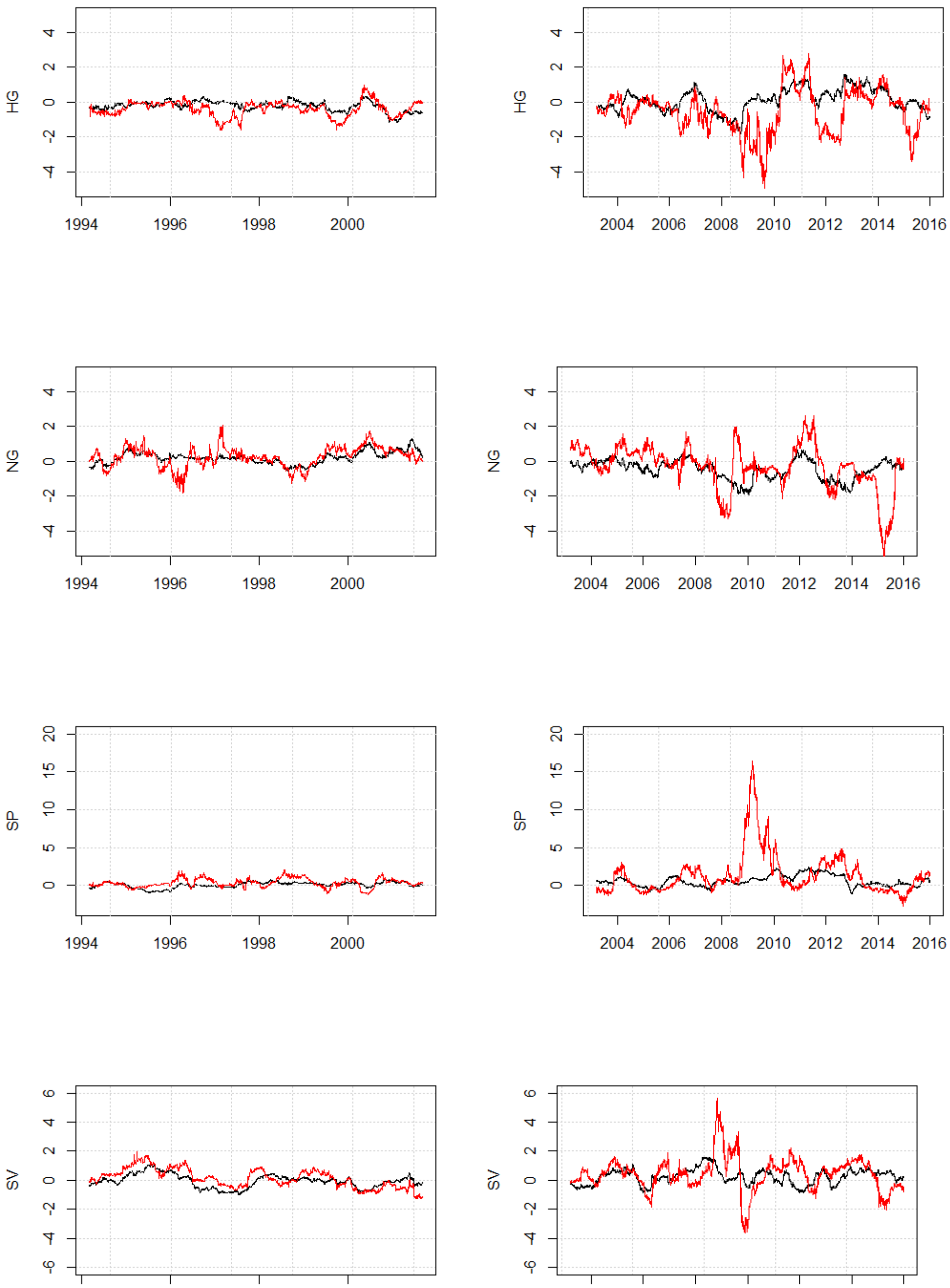
Figure 7: Directional NET spillovers computed as difference between TO and FROM spillovers. Red line presents NET directional volatility spillovers with persistence longer than two weeks. Black line shows NET connectedness with persistence shorter than two weeks.

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6 Conclusion

The analysis of volatility connectedness gives an inside into a risk transmission among markets which is useful for risk management and portfolio optimization as well as for accurate pricing of derivatives. The transfer of risk among commodities and equity may have implications for various participants on the market because number of investors is involved in both markets in order to diversify their portfolio. From the other point of view, authorities should carefully evaluate volatility transmission from stock markets to commodities, because the volatility of commodity prices may have a negative impact on global wealth.

In this paper I have given evidence of increasing interconnectedness of the commodity market as well as stronger volatility linkages of equity and commodities in the last decade, which is in line with the previous studies Diebold et al. (2017), Tang and Xiong (2012) and Malirova (2017) and supports hypothesis that the commodity market has been financialized. I enrich existing literature by analysing persistence in volatility connections, showing an increasing importance of long persistent volatility linkages on the overall connectedness of the commodity market. Why should I expect that short-run and long-run connectedness vary? The reason is that different agents on financial and commodity markets may operate on various investment horizons. Thus the insight into frequency dynamics helps to understand the nature of connections on a market, having variety of applications in risk management, for example in the VaR modelling.

The dynamic analysis reveals that level linkages among commodities and equity may vary a lot over the time. The stock market is a source of volatility especially in turbulent times. The highest volatility transmission from S&P to the commodity market was reached in 2007-2008 when more than 15 percent of volatility in commodities was explained by shocks in volatility from the equity market. Frequency dynamics of connectedness indicate that volatility spillovers from S&P to commodity market has mainly long-term nature and long-term volatility spillovers are associated with long-term price shocks. On the other hand, spillovers from the commodity market to the equity market were considerably lower and a half of them had short-run character with a persistence lower than two weeks. Among commodity sectors the energy market is the one with the largest impact on the stock market, showing large and long persistent volatility transmission from energies in 2014 when the prices of oil plunged.

The analysis gives a first inside into the persistence of volatility connections among different commodity sectors and equity, but there is still large space that is uncovered. Firstly, it would be great to use larger dataset which would shed light on the interconnectedness of commodity sectors. Our results in case of gold and silver suggest that spillovers within sectors may last shorter than connections between different sectors as the information flows are processed within the sector more quickly. However, I have a lack of evidence to confirm this hypothesis. Secondly, as volatility spillovers and price co-movements are linked, one may use our results to construct an appropriate hedging strategy and challenge strategies investigated in Arouri et al. (2011). Another possible application is to use our analysis for construction of VaR measure that would take into account persistence of connections. Thus there is a lot of space for application and extension of the results. Nevertheless, the methodology inherits drawbacks of the vector autoregressive model, thus a comparison with another models that produce a variance decomposition, for example non-parametric VAR models, would be welcomed.

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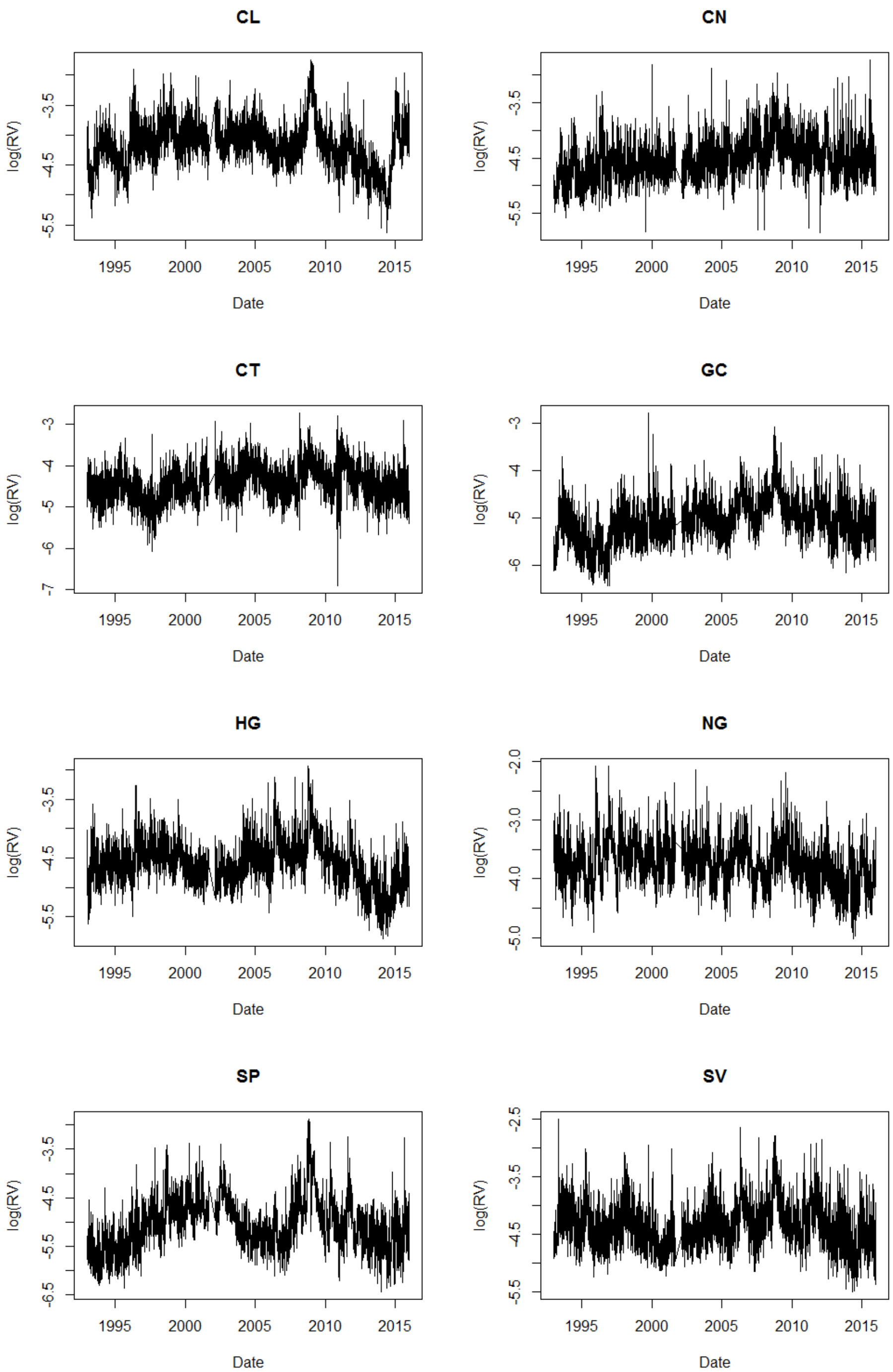
Appendix 1: Log realized volatility

Figure 8: Logarithm of realized volatilities



Source: Author's computations based on the data from TickData Inc.

Appendix 2: Gaussian Q-Q plots

Figure 9: Q-Q plots of realized volatilities versus quantiles of normal distribution

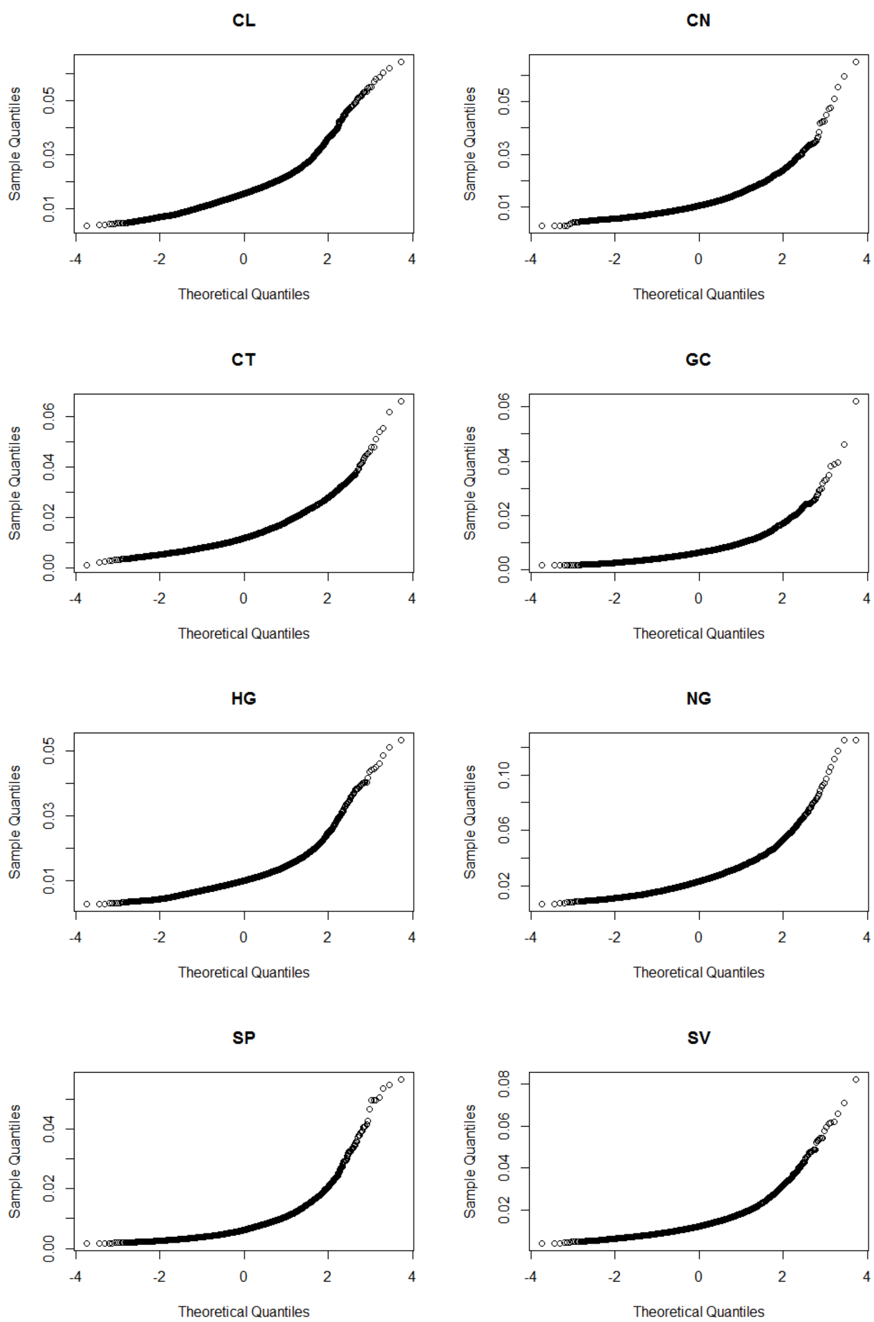
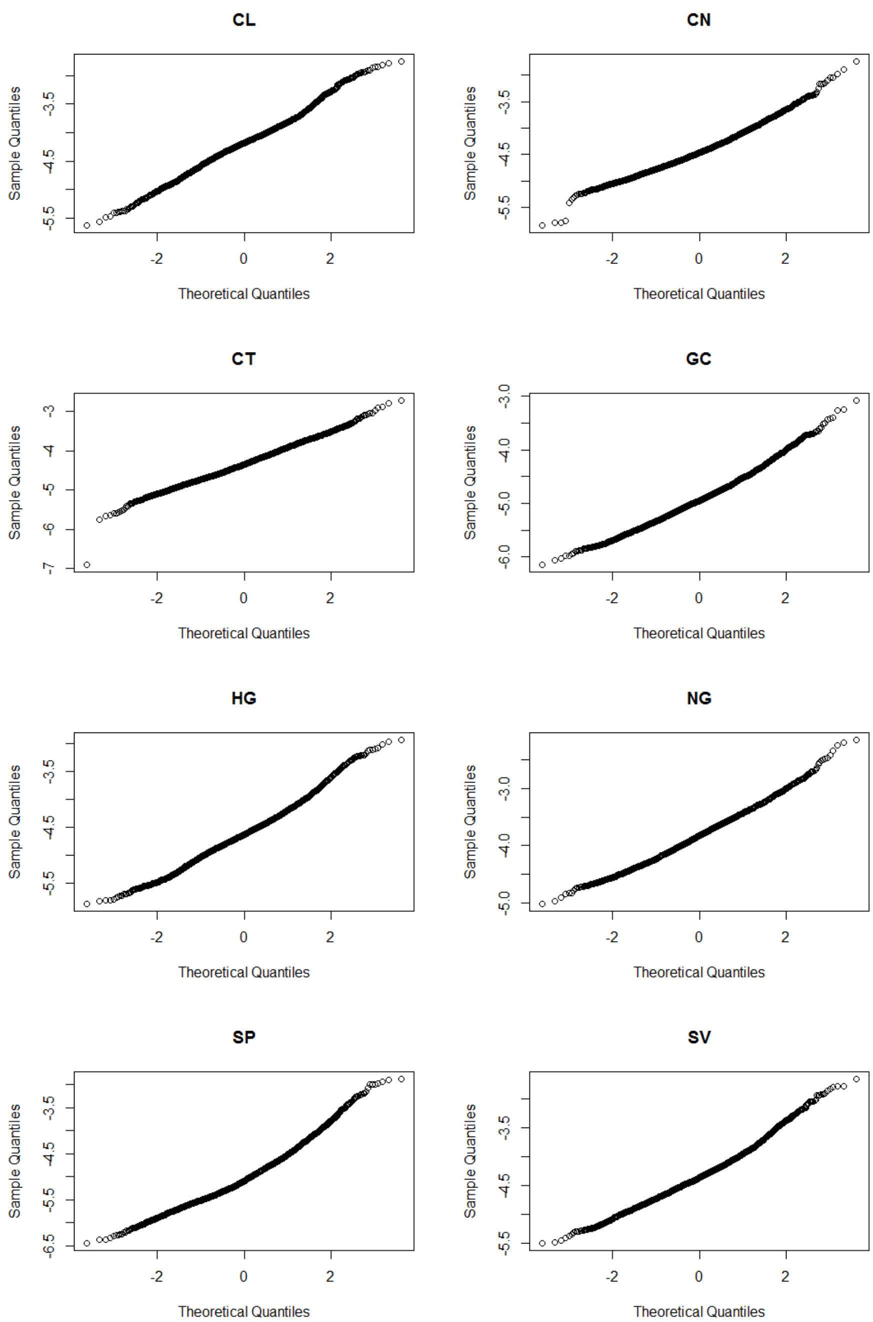


Figure 10: Q-Q plots log realized volatilities versus quantiles of normal distribution



Appendix 3: Connectedness matrices

Connectedness matrices estimated over the period 1993-2001

Table 7: Overall Connectedness Table, 1993-2001

	CL	CN	CT	GC	HG	NG	SP	SV
CL	0.819	0.013	0.001	0.017	0.012	0.013	0.121	0.004
CN	0.032	0.875	0.007	0.004	0.026	0.008	0.048	0.001
CT	0.003	0.002	0.946	0.001	0.023	0.017	0.005	0.003
GC	0.005	0.003	0.0003	0.737	0.006	0.001	0.068	0.179
HG	0.005	0.041	0.006	0.009	0.869	0.002	0.020	0.047
NG	0.022	0.003	0.015	0.002	0.005	0.923	0.024	0.006
SP	0.058	0.014	0.006	0.053	0.004	0.004	0.840	0.020
SV	0.010	0.001	0.003	0.222	0.020	0.0003	0.025	0.718

Table 8: Short-run Connectedness Table, 1993-2001

	CL	CN	CT	GC	HG	NG	SP	SV
CL	0.400	0.002	0.0005	0.003	0.003	0.004	0.008	0.002
CN	0.006	0.582	0.003	0.0005	0.007	0.003	0.004	0.0003
CT	0.002	0.002	0.604	0.0003	0.005	0.004	0.001	0.0003
GC	0.003	0.001	0.0001	0.397	0.005	0.0001	0.020	0.118
HG	0.004	0.012	0.002	0.007	0.532	0.001	0.010	0.017
NG	0.006	0.001	0.005	0.001	0.001	0.497	0.002	0.002
SP	0.003	0.001	0.001	0.013	0.003	0.001	0.251	0.009
SV	0.004	0.0005	0.001	0.141	0.011	0.0002	0.014	0.469

Table 9: Long-run Connectedness Table, 1993-2001

	CL	CN	CT	GC	HG	NG	SP	SV
CL	0.420	0.011	0.001	0.014	0.009	0.009	0.112	0.002
CN	0.026	0.293	0.004	0.004	0.018	0.006	0.044	0.001
CT	0.001	0.0002	0.342	0.0004	0.018	0.013	0.004	0.002
GC	0.002	0.003	0.0002	0.339	0.001	0.001	0.048	0.061
HG	0.002	0.030	0.004	0.002	0.336	0.001	0.010	0.031
NG	0.016	0.001	0.010	0.001	0.004	0.426	0.022	0.004
SP	0.055	0.013	0.006	0.040	0.002	0.003	0.589	0.012
SV	0.007	0.001	0.002	0.081	0.009	0.0001	0.012	0.248

Connectedness matrices estimated over the period 2002-2007

Table 10: Overall Connectedness Table, 2002-2007

	CL	CN	CT	GC	HG	NG	SP	SV
CL	0.806	0.003	0.015	0.042	0.036	0.048	0.017	0.032
CN	0.008	0.910	0.007	0.012	0.030	0.002	0.010	0.020
CT	0.015	0.008	0.897	0.022	0.015	0.031	0.007	0.004
GC	0.039	0.004	0.015	0.605	0.058	0.030	0.029	0.220
HG	0.017	0.006	0.013	0.080	0.678	0.006	0.049	0.151
NG	0.075	0.001	0.011	0.025	0.008	0.868	0.009	0.003
SP	0.007	0.006	0.002	0.019	0.038	0.009	0.889	0.029
SV	0.023	0.004	0.004	0.227	0.101	0.005	0.036	0.599

Table 11: Short-run Connectedness Table, 2002-2007

	CL	CN	CT	GC	HG	NG	SP	SV
CL	0.509	0.001	0.008	0.023	0.013	0.040	0.015	0.013
CN	0.002	0.637	0.004	0.005	0.005	0.001	0.002	0.004
CT	0.006	0.005	0.642	0.007	0.009	0.007	0.004	0.003
GC	0.017	0.003	0.005	0.361	0.021	0.010	0.019	0.122
HG	0.007	0.001	0.005	0.023	0.331	0.005	0.010	0.033
NG	0.061	0.001	0.002	0.009	0.006	0.506	0.007	0.003
SP	0.005	0.001	0.001	0.012	0.005	0.004	0.221	0.006
SV	0.011	0.002	0.002	0.123	0.027	0.004	0.012	0.358

Table 12: Long-run Connectedness Table, 2002-2007

	CL	CN	CT	GC	HG	NG	SP	SV
CL	0.297	0.003	0.006	0.019	0.023	0.008	0.002	0.018
CN	0.006	0.272	0.003	0.007	0.025	0.001	0.009	0.017
CT	0.009	0.003	0.255	0.015	0.006	0.023	0.003	0.002
GC	0.022	0.001	0.010	0.244	0.037	0.020	0.010	0.098
HG	0.009	0.005	0.008	0.057	0.347	0.001	0.039	0.118
NG	0.014	0.0002	0.009	0.016	0.001	0.362	0.001	0.001
SP	0.002	0.005	0.001	0.007	0.033	0.005	0.669	0.023
SV	0.012	0.002	0.002	0.104	0.075	0.002	0.024	0.241

Connectedness matrices estimated over the period 2007-2015

Table 13: Overall Connectedness Table, 2007-2015

	CL	CN	CT	GC	HG	NG	SP	SV
CL	0.559	0.020	0.020	0.051	0.129	0.020	0.147	0.055
CN	0.019	0.718	0.047	0.042	0.083	0.0005	0.048	0.042
CT	0.041	0.051	0.725	0.012	0.065	0.002	0.060	0.042
GC	0.051	0.018	0.006	0.413	0.135	0.013	0.116	0.247
HG	0.108	0.033	0.023	0.108	0.484	0.014	0.103	0.127
NG	0.086	0.003	0.006	0.034	0.065	0.750	0.035	0.021
SP	0.093	0.018	0.022	0.081	0.094	0.004	0.636	0.052
SV	0.056	0.022	0.023	0.247	0.156	0.008	0.074	0.415

Table 14: Short-run Connectedness Table, 2007-2015

	CL	CN	CT	GC	HG	NG	SP	SV
CL	0.154	0.002	0.002	0.014	0.015	0.005	0.016	0.013
CN	0.005	0.461	0.018	0.010	0.012	0.0001	0.009	0.009
CT	0.006	0.020	0.474	0.002	0.008	0.001	0.010	0.008
GC	0.020	0.004	0.001	0.220	0.035	0.006	0.032	0.122
HG	0.014	0.004	0.002	0.025	0.142	0.003	0.015	0.027
NG	0.019	0.0001	0.001	0.010	0.009	0.376	0.006	0.006
SP	0.013	0.003	0.003	0.022	0.014	0.002	0.150	0.013
SV	0.019	0.004	0.004	0.122	0.038	0.004	0.020	0.217

Table 15: Long-run Connectedness Table, 2007-2015

	CL	CN	CT	GC	HG	NG	SP	SV
CL	0.405	0.018	0.018	0.036	0.114	0.015	0.131	0.042
CN	0.013	0.257	0.028	0.032	0.071	0.0004	0.040	0.034
CT	0.035	0.031	0.252	0.010	0.058	0.001	0.050	0.034
GC	0.031	0.014	0.005	0.194	0.099	0.007	0.084	0.126
HG	0.094	0.029	0.021	0.083	0.342	0.012	0.087	0.100
NG	0.067	0.003	0.005	0.024	0.056	0.374	0.030	0.015
SP	0.080	0.016	0.019	0.059	0.080	0.001	0.487	0.038
SV	0.038	0.017	0.019	0.125	0.117	0.004	0.053	0.198